Program : **B.Tech**

Subject Name: **Fluid Mechanics**

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TRINITY

FLUID MECHANICS (ME – 404)

UNIT – 2 (KINEMATICS OF FLOW)

INTRODUCTION

A flow field is a region in which the flow is defined at all points at any instant of time. The means to that is to define the velocities at all the points at different times. It should be noted that the velocity at a point is the velocity of the fluid particle that occupies that point. In order to obtain a complete picture of the flow the fluid motion should be described mathematically. Just like the topography of a region is visualized using the contour map, the flow can be visualized using the velocity at all points at a given time or the velocity of a given particle at different times. It is then possible to also define the potential causing the flow.

Application of a shear force on an element or particle of a fluid will cause continuous deformation of the element. Such continuing deformation will lead to the displacement of the fluid element from its location and this results in fluid flow. The fluid element acted on by the force may move along a steady regular path or randomly changing path depending on the factors controlling the flow. The velocity may also remain constant with time or may vary randomly. In some cases the velocity may vary randomly with time but the variation will be about a mean value. It may also vary completely randomly as in the atmosphere. The study of the velocity of various particles in the flow and the instantaneous flow pattern of the flow field is called flow kinematics or hydrodynamics. Such a study is generally limited to ideal fluids, fluids which are incompressible and inviscid. In real fluid shows, beyond a certain distance from the surfaces, the flow behaves very much like ideal fluid. Hence these studies are applicable in real fluid flow also with some limitations.

LAGRANGIAN AND EULARIAN METHODS OF STUDY OF FLUID FLOW

In the Lagrangian method a single particle is followed over the flow field, the co-ordinate system following the particle. The flow description is particle based and not space based. A moving coordinate system has to be used. This is equivalent to the observer moving with the particle to study the flow of the particle. This method is more involved mathematically and is used mainly in special cases.

In the Eularian method, the description of flow is on fixed coordinate system based and the description of the velocity etc. are with reference to location and time *i.e.*, $V = V(x, y, z, t)$ and not with reference to a particular particle. Such an analysis provides a picture of various parameters at all locations in the flow field at different instants of time. This method provides an easier visualisation of the flow field and is popularly used in fluid flow studies. However the final description of a given flow will be the same by both the methods.

BASIC SCIENTIFIC LAWS USED IN THE ANALYSIS OF FLUID FLOW

Law of conservation of mass: This law when applied to a control volume statesthat the net mass flow through the volume will equal the mass stored or removed from the volume. Under conditions of steady flow this will mean that the mass leaving the control volume should be equal to the mass entering the volume. The determination of flow velocity for a specified mass flow rate and flow area is based on the continuity equation derived on the basis of this law.

Newton's laws of motion: These are basic to any force analysis under variousconditions of flow. The resultant force is calculated using the condition that it equals the rate of change of momentum. The reaction on surfaces are calculated on the basis of these laws. Momentum equation for flow is derived based on these laws.

Law of conservation of energy: Considering a control volume the law can bestated as "the energy flow into the volume will equal the energy flow out of the volume under steady conditions". This also leads to the situation that the total energy of a fluid element in a steady flow field is conserved. This is the basis for the derivation of Euler and Bernoulli equations for fluid flow.

Thermodynamic laws: are applied in the study of flow of compressible fluids.

FLOW OF IDEAL / INVISCID AND REAL FLUIDS

Ideal fluid is no viscous and incompressible. Shear force between the boundary surface and fluid or between the fluid layers is absent and only pressure forces and body forces are controlling.

Real fluids have viscosity and surface shear forces are involved during flow. However the flow after a short distance from the surface is not affected by the viscous effects and approximates to ideal fluid flow. The results of ideal fluid flow analysis are found applicable in the study of flow of real fluids when viscosity values are small.

STEADY AND UNSTEADY FLOW

In order to study the flow pattern it is necessary to classify the various types of flow. The classification will depend upon the constancy or variability of the velocity with time. In the next three sections, these are described. In steady flow the property values at a location in the flow are constant and the values do not vary with time. The velocity or pressure at a point remains constant with time. These can be expressed as $V = V(x, y, z)$, $P = P(x, y, z)$ etc. In steady flow a picture of the flow field recorded at different times will be identical. In the case of unsteady flow, the properties vary with time or $V = V(x, y, z, t)$, $P = P(x, y, z, t)$ where t is time.

In unsteady flow the appearance of the flow field will vary with time and will be constantly changing. In turbulent flow the velocity at any point fluctuates around a mean value, but the mean value at a point over a period of time is constant. For practical purposes turbulent flow is considered as steady flow as long as the mean value of properties do not vary with time.

COMPRESSIBLE AND INCOMPRESSIBLE FLOW

If the density of the flowing fluid is the same all over the flow field at all times, then such flow is called incompressible flow. Flow of liquids can be considered as incompressible even if the density varies a little due to temperature difference between locations. Low velocity flow of gases with small changes in pressure and temperature can also be considered as incompressible flow. Flow through fans and blowers is considered incompressible as long as the density variation is below 5%. If the density varies with location, the flow is called compressible flow. In this chapter the study is mainly on incompressible flow.

LAMINAR AND TURBULENT FLOW

If the flow is smooth and if the layers in the flow do not mix macroscopically then the flow is called laminar flow. For example a dye injected at a point in laminar flow will travel along a continuous smooth line without generally mixing with the main body of the fluid. Momentum, heat and mass transfer between layers will be at molecular level of pure diffusion. In laminar flow layers will glide over each other without mixing.

In turbulent flow fluid layers mix macroscopically and the velocity/temperature/mass concentration at any point is found to vary with reference to a mean value over a time period. For example $u = \bar{u} + u'$

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where *u* is the velocity at an instant at a location and $\bar{u}$ is the average velocity over a period of time at that location and *u*′ is the fluctuating component. This causes higher rate of momentum/heat/mass transfer. A dye injected into such a flow will not flow along a smooth line but will mix with the main stream within a short distance.

The difference between the flows can be distinguished by observing the smoke coming out of an incense stick. The smoke in still air will be found to rise along a vertical line without mixing. This is the laminar region. At a distance which will depend on flow conditions the smoke will be found to mix with the air as the flow becomes turbulent. Laminar flow will prevail when viscous forces are larger than inertia forces. Turbulence will begin where inertia forces begin to increase and become higher than viscous forces.

CONCEPTS OF UNIFORM FLOW, REVERSIBLE FLOW AND THREE DIMENSIONAL FLOW

If the velocity value at all points in a flow field is the same, then the flow is defined as uniform flow. The velocity in the flow is independent of location. Certain flows may be approximated as uniform flow for the purpose of analysis, though ideally the flow may not be uniform.

If there are no pressure or head losses in the fluid due to frictional forces to be overcome by loss of kinetic energy (being converted to heat), the flow becomes reversible. The fluid can be restored to its original condition without additional work input. For a flow to be reversible, no surface or fluid friction should exist. The flow in a venturi (at low velocities) can be considered as reversible and the pressures upstream and downstream of the venturi will be the same in such a case. The flow becomes irreversible if there are pressure or head losses.

If the components of the velocity in a flow field exists only in one direction it is called one dimensional flow and $V = V(x)$. Denoting the velocity components in *x*, *y* and *z* directions as *u*, *v* and *w*, in one dimensional flow two of the components of velocity will be zero. In twodimensional flow one of the components will be zero or $V = V(x, y)$. In three dimensional flow all the three components will exist and $V = V(x, y, z)$. This describes the general steady flow situation. Depending on the relative values of *u*, *v* and *w* approximations can be made in the analysis. In unsteady flow $V = V(x, y, z, t)$.

VELOCITY AND ACCELERATION COMPONENTS

Two basic and important field variables in the study of fluid mechanics are the velocity and acceleration of the fluid, and they are the focus of the discussion in this section. Both the velocity and acceleration equations are presented in Eulerian viewpoint.

VELOCITY FIELD

Velocity is an important basic parameter governing a flow field. Other field variables such as the pressure and temperature are all influenced by the velocity of the fluid flow. In general, velocity is a function of both the location and time. The velocity vector can be expressed in Cartesian coordinates as

$$V = V(x,y,z,t)$$
$$= u(x,y,z,t)\, i + v(x,y,z,t) j + w\,(x,y,z,t)\, k$$

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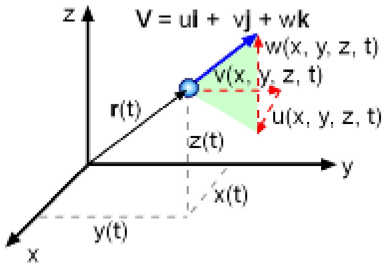


Figure: Velocity Component

Where the velocity components (u, v and w) are functions of both position and time. That is, u = u(x, y, z, t), v = v(x, y, z, t) and w = w(x, y, z, t). This velocity description is called the velocity field since it describes the velocity of all points, Eulerian viewpoint, in a given volume.

For a single particle, Lagrangian viewpoint, the velocity is derived from the changing position vector, or

V = d**r**/dt = = dx/dt **i** + dy/dt **j** + dz/dt **k**

Where **r** is the position vector (**r** = x**i** + y**j** + z**k**). Notice, the velocity is only a function of time since it is only tracking a single particle.

ACCELERATION FIELD

Another important parameter in the study of fluid in motion is the acceleration. Acceleration is related to the velocity, and it can be determined once the velocity field is known. The acceleration is the change in velocity, δ**V**, over the change in time, δt,

a = [d**V**(t + δ) - d**V**(t)] / δt = δ**V**/δt = d**V**/dt

But it is not just a simple derivative of just time since the velocity is a function time, AND space (x,y,z). The change in velocity must be track in both time and space. Using the chain rule of calculus, the change in velocity is,

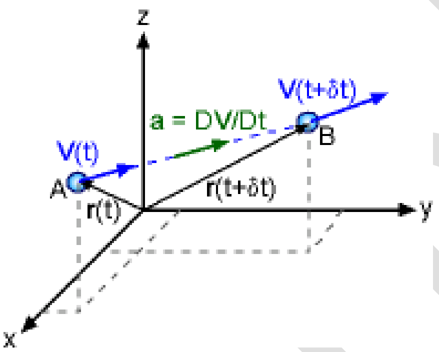


Figure: Acceleration Field

$$d\mathbf{V} = \frac{\partial \mathbf{V}}{\partial x}dx + \frac{\partial \mathbf{V}}{\partial y}dy + \frac{\partial \mathbf{V}}{\partial z}dz + \frac{\partial \mathbf{V}}{\partial t}dt$$

Or

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \mathbf{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \mathbf{V}}{\partial z}\frac{dz}{dt} + \frac{\partial \mathbf{V}}{\partial t}$$

This can simplified using u, v, and w, the velocity magnitudes in the three coordinate directions. In Cartesian coordinates, the acceleration field is:

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$$

This expression can be expanded and rearranged as

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

The acceleration equation can also be written in polar coordinates and are given in the Basic Equations appendix.

MATERIAL DERIVATIVE

The time and space derivative used to determine the acceleration field from the velocity is so common in fluid mechanics, it has a special name. It is called the Material or Substantial Derivative and has a special symbol, D ()/Dt. For Cartesian coordinates, it is

$$\frac{D(\;)}{Dt} = \frac{\partial(\;)}{\partial t} + u\frac{\partial(\;)}{\partial x} + v\frac{\partial(\;)}{\partial y} + w\frac{\partial(\;)}{\partial z}$$

Or in vector form,

$$\frac{D(\;)}{Dt} = \frac{\partial(\;)}{\partial t} + \mathbf{V}\cdot\nabla(\;)$$

Where ∇ is the gradient operator from the above equation, it can be seen that the material derivative consists of two terms. The first term ∂t ( )/∂t is referred to as the local rate of change, and it represents the effect of unsteadiness. For steady flow, the local derivative vanishes (i.e., ∂t ( )/∂t = 0).

The second term, $\mathbf{V}\cdot\nabla(\;)$, is referred to as the convective rate of change, and it represents the variation due to the change in the position of the fluid particle, as it moves through a field with a gradient. If there is no gradient (no spatial change) then $\nabla(\;)$ is zero so there is no convective change.As an example, the acceleration field equation can be written as

$$a = \frac{D\mathbf{V}}{Dt} = \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}$$

CONTINUITY EQUATION FOR FLOW—CARTESIAN CO-ORDINATES

The continuity equation is an expression of a fundamental conservation principle, namely, that of mass conservation. It is a statement that fluid mass is conserved: all fluid particles that flow into any fluid region must flow out. To obtain this equation, we consider a cubical control volume inside a fluid. Mass conservation requires that the net flow through the control volume is zero. In other words, all fluid that is accumulated inside the control volume (due to compressibility for example) + all fluid that is flowing into the control volume must be equal to the amount of fluid flowing out of the control volume. Accumulation + Flow In = Flow Out.

Consider a fluid element of lengths dx, dy, dz in the direction of x, y, z.

Let u, v, w are the inlet velocity components in x, y, z direction respectively.

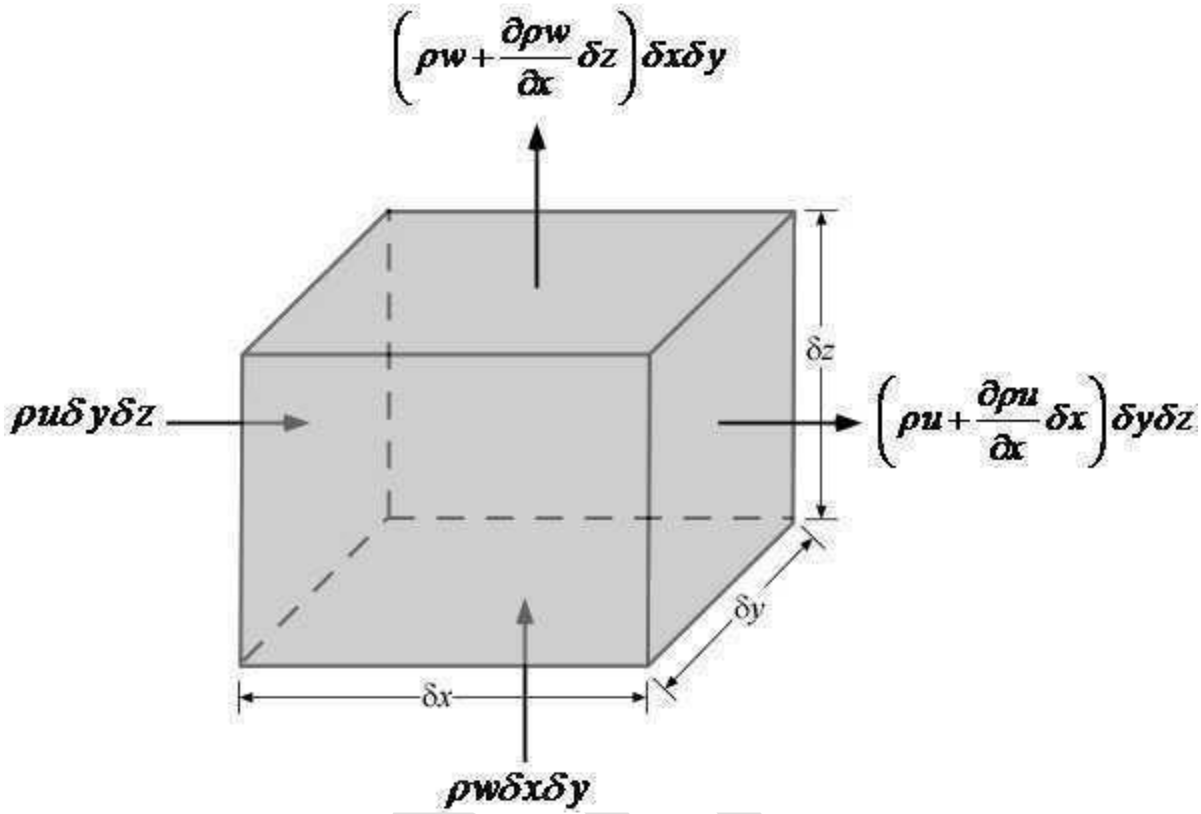
Let, p is mass density of fluid element at particular instate.

Mass of fluid entering the face ABCD (In flow)= Mass density x Velocity x-direction x area of ABCD

= p x u x (∂y x ∂z)

Then mass of fluid leaving the face EFGH (out flow) = (pu ∂y. ∂z) + ∂ / ∂x (pu∂y∂z)

Rate of increases in mass x-direction = Outflow – Inflow

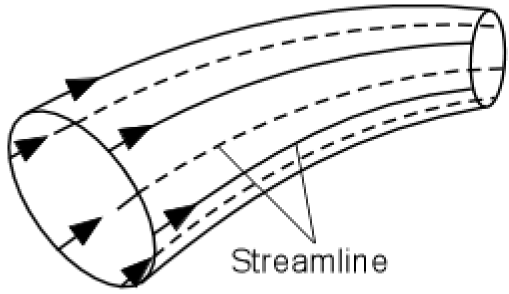


= [(pudydx) + ∂ / ∂_x (pudydz) d_x] - (pudydz)
Rate of increases in mass x direction = ∂ / ∂_x p u d_x d_y d_z
Similarly,
Rate of increase in mass y-direction = ∂ / ∂y p v ∂x ∂y ∂z
Rate of increases in mass z-direction = ∂ / ∂_z p w ∂_x ∂y ∂_z
Total rate of increases in mass = $\partial_x \partial_y \partial_z$ [∂p u / ∂_x + ∂p v / ∂_y + ∂p w / ∂_z]
By law of conservation of mass, there is no accumulation of mass, and hence the above quantity must be zero.
∂x. ∂y. ∂z [∂pu / ∂_x + ∂pv / ∂_y + ∂pw / ∂_z] = 0
∂ (pu) / ∂_x +∂ (pv) / ∂_y + ∂ (pw) / ∂_z = 0 (for compressible fluid)
If fluid is incompressible, then is constant
$\partial_u / \partial_x + \partial_v / \partial_y + \partial_w / \partial_z = 0$
This is the continuity equation for three-dimensional flow.

STREAM LINES, STREAM TUBE, PATH LINES, STREAK LINES ANDTIME LINES

The analytical description of flow velocity is geometrically depicted through the concept of stream lines. The velocity vector is a function of both position and time. If at a fixed instant of time a curve is drawn so that it is tangent everywhere to the velocity vectors at these locations then the curve is called a stream line. Thus stream line shows the mean direction of a number of particles in the flow at the same instant of time. **Stream lines are a series of curvesdrawn tangent to the mean velocity vectors of a number of particles in the flow. Since stream lines are tangent to the velocity vector at every point in the flow field, there can be no flow across a stream line.**

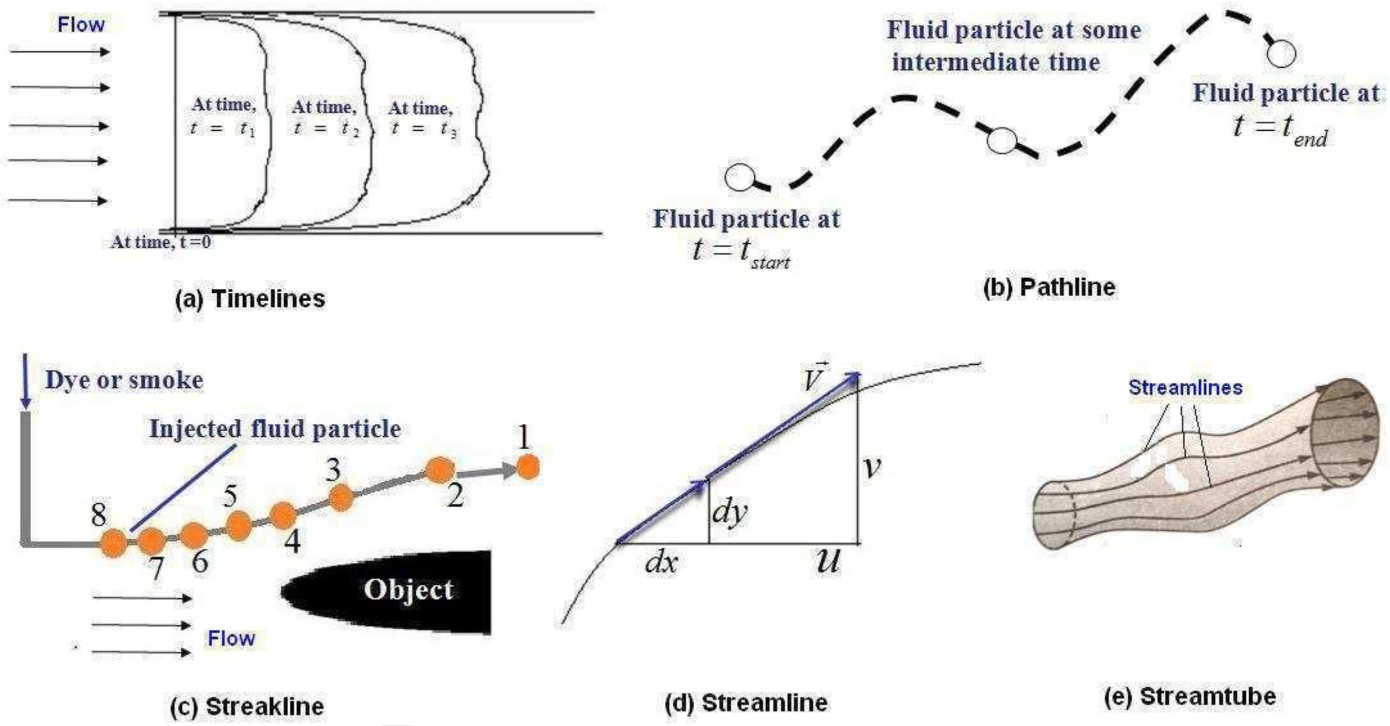
A bundle of neighboring stream lines may be imagined to form a passage through which the fluid flows. Such a passage is called a stream tube. Since the stream tube is bounded on all sides by stream lines, there can be no flow across the surface. Flow can be only through the ends.



Under steady flow condition, the flow through a stream tube will be constant along the length.

Path line is the trace of the path of a single particle over a period of time. Pathline shows the direction of the velocity of a particle at successive instants of time. In steady flow path lines and stream lines will be identical.

Streak lines provide an instantaneous picture of the particles, which have passed through a given point like the injection point of a dye in a flow. In steady flowthese lines will also coincide with stream lines.



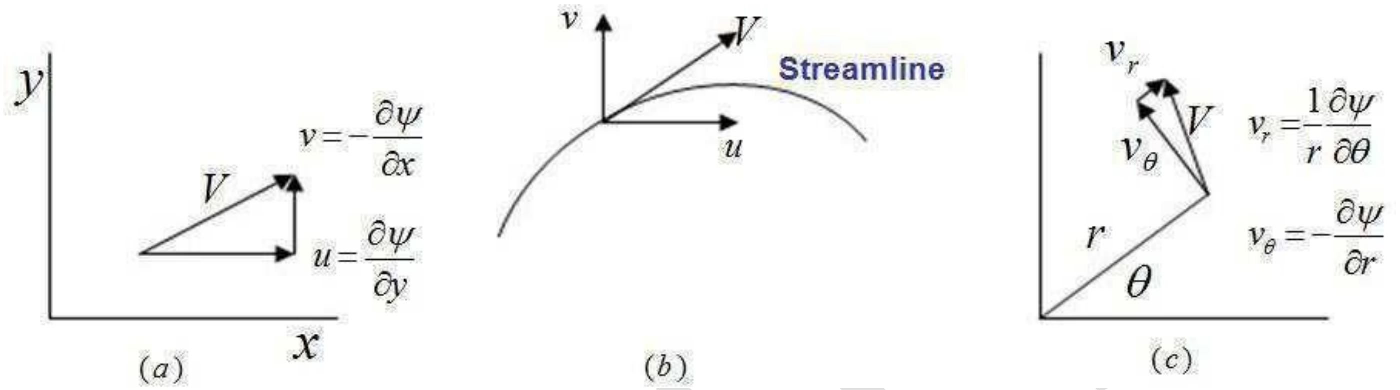
Particles P_1, P_2, P_3, P_4, starting from point P at successive times pass along path lines shown. At the instant of time considered the positions of the particles are at 1, 2, 3 and 4. A line joining these points is the streak line.

If a number of adjacent fluid particles in a flow field are marked at a given instant, they form a line at that instant. This line is called time line. Subsequent observations of the line may provide information about the flow field. For example the deformation of a fluid under shear force can be studied using time lines.

CONCEPT OF STREAM LINE

In a flow field if a continuous line can be drawn such that the tangent at every point on the line gives the direction of the velocity of flow at that point, such a line is defined as a stream line. In steady flow any particle entering the flow on the line will travelonly along this line. This leads to visualisation of a stream line in laminar flow as the path of a dye injected into the flow.

There can be no flow across the stream line, as the velocity perpendicular to thestream line is zero at all points. The flow along the stream line can be considered as one dimensional flow, though the stream line may be curved as there is no component of velocity in the other directions. Stream lines define the flow paths of streams in the flow. The flow entering between two stream lines will always flow between the lines. The lines serve as boundaries for the stream.



CONCEPT OF STREAM FUNCTION

Stream function is a very useful device in the study of fluid dynamics and was arrived at by the French mathematician Joseph Louis Lagrange in 1781. Of course, it is related to the streamlines of flow, a relationship which we will bring out later. We can define stream functions for both two and three dimensional flows. The latter one is quite complicated and not necessary for our purposes. We restrict ourselves to two-dimensional flows.

Consider a two-dimensional incompressible flow for which the continuity equation is given by,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

A stream function is one which satisfies

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) \equiv 0$$

VELOCITY POTENTIAL

We have seen that for an irrotational flow

$$curl(\vec{V}) = 0 \quad or \quad \nabla \times \vec{V} = 0$$

It follows from vector algebra that there should be a potential such that:

$$\vec{V} = \nabla \phi$$

Φ is called the Velocity Potential. The velocity components are related to ϕ through the following relations.

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{\partial \phi}{\partial z}$$

Velocity potential is a powerful tool in analyzingirrotational flows. First of all it meets with the irrotationality condition readily. In fact, it follows from that condition. As a check we substitute the velocity potential in the irrotationality condition, thus,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}\frac{\partial \phi}{\partial y} - \frac{\partial}{\partial y}\frac{\partial \phi}{\partial x} \equiv 0$$

The next question we ask is does the velocity potential satisfy the continuity equation? To find out we consider the continuity equation for incompressible flows and substitute the expressions for velocity coordinates in them. Accordingly,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\frac{\partial \phi}{\partial x} + \frac{\partial}{\partial y}\frac{\partial \phi}{\partial y} = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}$$

It is clear that to meet with the continuity requirements the velocity potential has to satisfy the equation,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

In vector notation it is

$$\nabla^2 \phi = 0$$

As with stream functions we can have lines along which potential ϕ is constant. These are called Equipotential Lines of the flow. Thus along a potential line ϕ = C.

The equation is called the Laplace Equation and is encountered in many branches of physics and engineering. A flow governed by this equation is called a Potential Flow. Further the Laplace equation is linear and is easily solved by many available standard techniques, of course, subject to boundary conditions at the boundaries.

RELATIONSHIP BETWEEN Φ AND Ψ

1. We notice that velocity potential **Φ** and stream function **Ψ** are connected with velocity components. It is necessary to bring out the similarities and differences between them.

   Stream function is defined in order that it satisfies the continuity equation readily. We do not know yet if it satisfies the irrotationality condition. So we test out below. Recall that the velocity components are given by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

   Substituting these in the irrotationality condition, we have

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\frac{\partial \psi}{\partial y} = -\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}$$

   Which leads to the condition that $\nabla^2\psi = 0$ for irrotationality

   Thus we see that the velocity potential ϕ automatically complies with the irrtotatioanlity condition, but to satisfy the continuity equation it has to obey that $\nabla^2\psi = 0$. On the other hand the stream function readily satisfies the continuity condition, but to meet with the irrotationality condition it has to obey $\nabla^2\psi = 0$.

   Thus we see that the streamlines too follow the Laplace Equation. So it is possible to solve for a potential flow in terms of stream function.

Table: Properties of stream function and velocity potential

Property	ψ	ϕ
Continuity Equation	Automatically Satisfied	satisfied if $\nabla^2\psi = 0$
Irrotationality Condition	satisfied if $\nabla^2\psi = 0$	Automatically Satisfied

2. Streamlines and equipotential lines are orthogonal to each other. We have seen that the velocity components of the flow are given in terms of velocity potential and stream function by the equations,

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

   Those familiar with Complex Variables theory will recognize that these are the Cauchy-Riemann equations and that ϕ = C and ψ = D are orthogonal and that both ϕ and ψ obey Laplace Equation. However, we will prove the orthogonality condition by other means.

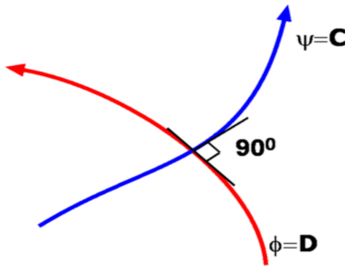


Figure: Orthogonality of Stream lines and equipotential lines

Since ϕ = C, it follows that

$$= \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

= 0

$$= u\ dx + v\ dy$$

The gradient of the equipotential line is hence given by

$$\left(\frac{dy}{dx}\right)_{\phi = C} = -\frac{u}{v}$$

On the other hand the gradient of a stream line is given by

$$\left(\frac{dy}{dx}\right)_{\psi = D} = \frac{v}{u}$$

Thus we find that

$$\left(\frac{dy}{dx}\right)_{\phi = C} \left(\frac{dy}{dx}\right)_{\psi = D} = -1$$

Showing that equipotential lines and streamlines are orthogonal to each other. This enables one to calculate the stream function when the velocity potential is given and vice versa.

Figure shows the flow through a bend where the streamlines and the equipotential lines have been plotted. The two form an orthogonal network.

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Figure: Stream lines and equipotential lines for flow through a bend

Uniform Flow

Uniform flow is the simplest form of potential flow. For flow in a specific direction, the velocity potential is

$\phi = U\,(x \cos\alpha + y \sin\alpha)$

While the stream function is$\psi = U\,(y \cos\alpha - x \sin\alpha)$

Where α represents the angle between the flow direction and the x-axis (as shown in the figure). Recall that the velocity potential and stream functionare related to the component velocity in the 2 dimensional flow field as follows:

Cartesian coordinates:

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

And

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

Cylindrical coordinates:

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

And

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

Note that the lines of the constant velocity potential (equipotential lines) are orthogonal to the lines of the stream function (streamlines).

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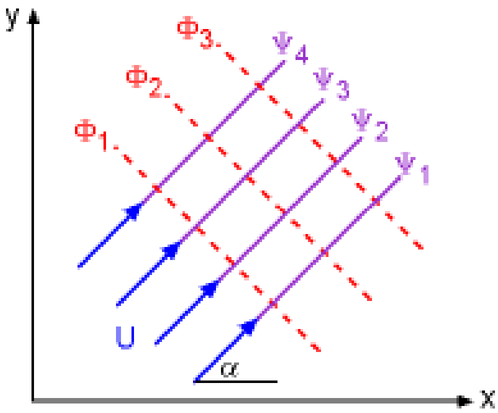

Figure: Uniform Flow

Source and Sink

When a fluid flows radially outward from a point source, the velocities are

$v_r = m / (2\pi r)$ and $v_\theta = 0$

Where m is the volume flow rate from the line source per unit length. The velocity potential and stream function can then be represented as:

$$\varphi = \frac{m}{2\pi}\ln(r) \qquad \psi = \frac{m}{2\pi}\theta$$

When m is negative, the flow is inward, and it represents a sink. The volume flow rate per unit depth, m, indicates the strength of the source or sink. Note that as r approaches zero, the radial velocity goes to infinity. Hence, the origin represents a singularity. As shown in the figure, the equipotential lines are given by the concentric circles while the streamlines are the radial lines.

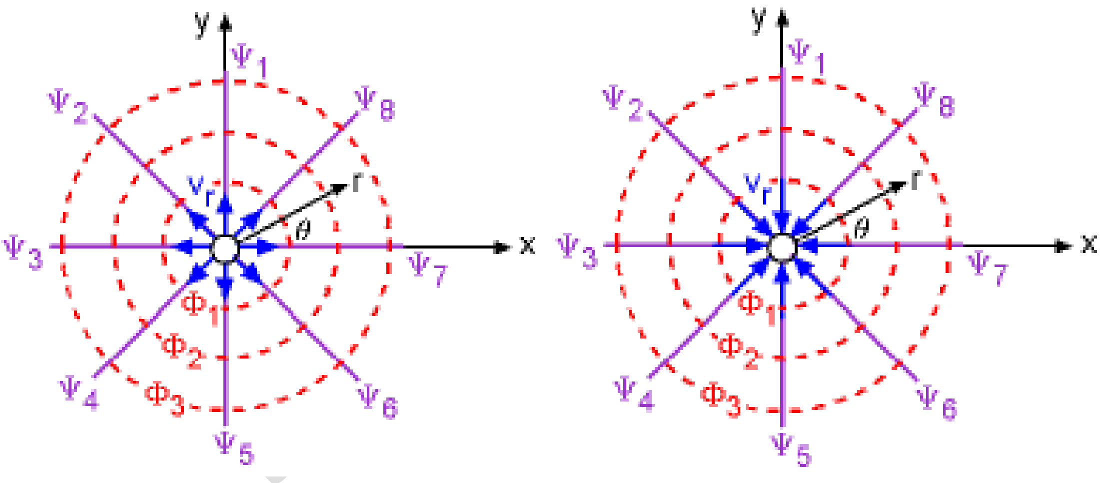

Figure: Source and Sink Flow

Vortex

A vortex can be obtained by reversing the velocity potential and stream functions for a point source such that $\phi = K\theta$ and $\psi = -K\ln(r)$

Where K is a constant indicating the strength of the vortex. Now, the equipotential lines are radial lines while the streamlines are given by the concentric circles. The velocities of a vortex are, $V_r = 0$.

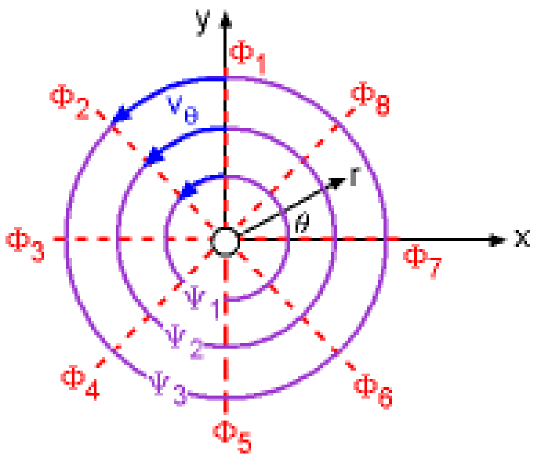

Figure: Vortex Flow

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