Program : **B.Tech**

Subject Name: **Machine Component Design**

Subject Code: **ME-602**

Semester: **6th**

ME-602 Machine Component and Design

Syllabus

Unit 1:

Introduction to stress in machine component: Stress concentration and fatigue: causes of stress concentration; stress concentration in tension, bending and torsion; reduction of stress concentration, theoretical stress concentration factor, notch sensitivity, fatigue stress concentration factor, cyclic loading, endurance limit, S-N Curve, loading factor, size factor, surface factor. Design consideration for fatigue, Goodman and modified Goodman's diagram, Soderberg equation, Gerber parabola, design for finite life, cumulative fatigue damage factor.

Subject Notes

Definition

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time-consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Machine Design Considerations

Following are the general considerations in designing a machine component:

1. **Type of load and stresses caused by the load - T**he load, on a machine component, may act in several ways due to which the internal stresses are set up.
2. **The motion of the parts or kinematics of the machine -** The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required. The motion of the parts may be:
   a) The rectilinear motion which includes unidirectional and reciprocating motions.
   b) The curvilinear motion which includes rotary, oscillatory and simple harmonic.
   c) Constant velocity.
   d) Constant or variable acceleration.
3. **Selection of materials -** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behavior under working conditions. Some of the important characteristics of materials are strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc.
4. **Form and size of the parts -** The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.
5. **Frictional resistance and lubrication -** There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. **Convenient and economical features -** In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take-up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. **Use of standard parts -** The use of standard parts is closely related to cost because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts, and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers, and taps and also to decrease the number of wrenches required.
8. **Safety of operation -** Some machines are dangerous to operate, especially those which are speeded up to ensure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with the operation of the machine.
9. **Workshop facilities -** A design engineer should be familiar with the limitations of his employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.
10. **Number of machines to be manufactured -** The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for a small number of the product will not permit any undue expense in the workshop processes so that the designer should restrict his specification to standard parts as much as possible.
11. **Cost of construction -** The cost of construction of an article is the most important consideration involved in the design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further consideration. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.
12. **Assembling -** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

Machine Design Procedure

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

1. **Recognition of need.** First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.
2. **Synthesis (Mechanisms).** Select the possible mechanism or group of mechanisms which will give the desired motion.
3. **Analysis of forces.** Find the forces acting on each member of the machine and the energy transmitted by each member.
4. **Material selection.** Select the material best suited for each member of the machine.

5. **Design of elements (Size and Stresses).** Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.
6. **Modification.** Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.
7. **Detailed drawing.** Draw the detailed drawing of each component and the assembly of the machine with the complete specification for the manufacturing processes suggested.
8. **Production.** The component, as per the drawing, is manufactured in the workshop.

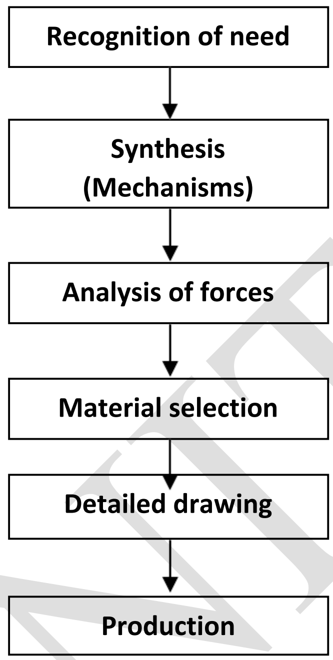


Fig. 1.1 Machine Design Procedure

Material selection
The engineering materials are mainly classified as:
1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.
2. Non-metals, such as glass, rubber, plastic, etc. The metals may be further classified as:
(a) Ferrous metals, and (b) Non-ferrous metals.
The **ferrous metals** are those which have the iron as their main constituents, such as cast iron, wrought iron, and steel.
The **non-ferrous** metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.
The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:
1. Availability of the materials,
2. Suitability of the materials for the working conditions in service, and
3. The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical properties.

Physical Properties of Metals

The physical properties of the metals include luster, color, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. **Strength.** It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.
2. **Stiffness.** It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.
3. **Elasticity.** It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.
4. **Plasticity.** It is the property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.
5. **Ductility.** It is the property of a material enabling it to be drawn into a wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminum, nickel, zinc, tin and lead.
6. **Brittleness.** It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.
7. **Malleability.** It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper, and aluminum.
8. **Toughness.** It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed to the point of fracture. This property is desirable in parts subjected to shock and impact loads.
9. **Machinability.** It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.
10. **Resilience.** It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.
11. **Creep.** When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers, and turbines.
12. **Fatigue.** When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue**. The failure is caused by means of a progressive crack formation which is usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.
13. **Hardness.** It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are

dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

i. Brinell hardness test,
ii. Rockwell hardness test,
iii. Vickers hardness (also called Diamond Pyramid) test, and
iv. Shore scleroscope

Effect of Impurities on Steel

The following are the effects of impurities like silicon, sulfur, manganese, and phosphorus on steel.

1. Manganese (Mn) – Manganese improves hardenability, ductility and wears resistance. Mn eliminates the formation of harmful iron sulphides, increasing strength at high temperatures.

2. Nickel (Ni) – Nickel increases strength, impact strength and toughness, impart corrosion resistance in combination with other elements.

3. Chromium (Cr) – Chromium improves hardenability, strength and wear resistance, sharply increases corrosion resistance at high concentrations (> 12%).

4. Tungsten (W) – Tungsten increases hardness particularly at elevated temperatures due to stable carbides, refines grain size.

5. Vanadium (V) – Vanadium increases strength, hardness, creep resistance and impact resistance due to the formation of hard vanadium carbides, limits grain size.

6. Molybdenum (Mo) – Molybdenum increases hardenability and strength particularly at high temperatures and under dynamic conditions.

7. Silicon (Si) – Silicon improves strength, elasticity, acid resistance and promotes large grain sizes, which cause increasing magnetic permeability.

8. Titanium (Ti) – Titanium improves strength and corrosion resistance, limits austenite grain size.

9. Cobalt (Co) – Cobalt improves strength at high temperatures and magnetic permeability.

10. Zirconium (Zr) – Zirconium increases strength and limits grain sizes.

11. Boron (B) – Boron highly effective hardenability agent, improves deformability and machinability.

12. Copper (Cu) – Copper improves corrosion resistance.

13. Aluminium (Al) – Aluminium acts as a deoxidizer, limits austenite grain growth.

Modes of failure –

It is the basic manner or mechanism of the failure or damage process. Failure is not performing as intended, may occur at any stage/action during manufacture or in-service due to one or more causes.

Some basic knowledge of modes of failure is needed for:

i. Modern design methods e.g. latest Standards aim at designing against each of the modes of failure which may be feasible for a particular component or equipment and service conditions.
ii. Manufacture and testing where differences may be required for virtually the same product when different materials or service conditions apply e.g. low temperature (brittle fracture), extensive cyclic loading (fatigue), or seawater (corrosion).
iii. Operation of equipment to be within design limits, but if these are to be exceeded the likelihood of failure by different modes can be seriously increased.
iv. Risk management of critical equipment where assessments are made of the hazards, feasible failure modes, the likelihood of failure & consequences e.g. FMEA or FMECA (Failure Modes Effects & Consequence Analysis).
v. In-service inspection to assess acceptability, avoidance or rectification of any degradation.
vi. Failure analysis to help determine and identify probable failure causes, methods of avoiding repetition and possible improvements by innovation.
vii. Training of technologists and key technical people on the “why” of various practices aimed at achieving safety of people and plant and protection of the environment through prevention of failures.

Various Modes of Failure are: -

1. Creep

a) Distortion **S**

b) Cracking **U**
c) Rupture (through wall) **U**
d) Creep-fatigue **U**
e) Creep buckling

2. **Brittle Fracture** U
a) Ferritic steel at notches at low temperature
b) Low ductility Material: Cast iron, glass etc

3. **Fatigue U**
a) Mechanical fatigue (high/low cycle)
b) Thermal fatigue
c) See also corrosion fatigue

4. **Excessive Deformation (elastic or plastic) S or U**
a) Thermal distortion by overheating
b) Overloading e.g. by over pressurization, overload, (earthquakes, earth settlement, and wind).
c) Causing leakage at mechanical joints.
d) Ratcheting incremental collapse – progressive (plastic deformation)
e) Mechanical gouging

5. **Ductile Fracture U**
a) Rupture following excessive plastic deformation. (Includes ductile tearing).
b) Lamellar tearing

6. **8.Instability (Buckling) U**
a) Elastic
b) Plastic or elastic – plastic
c) Creep buckling
d) Overturning

7. **Sustained Load Cracking U**
a) In 6000 series aluminum alloys

8. **Combinations of Above S or U**
a) Mixed modes e.g. corrosion & fatigue leading to brittle fracture; or corrosion-creep-fatigue.

U = Ultimate or Strength limit state
S = Serviceability limit state

Theories of failure: -

It has already been discussed in the previous chapter that strength of machine members is based upon the mechanical properties of the materials used. Since these properties are usually determined from simple tension or compression tests, therefore, predicting failure in members subjected to uniaxial stress is both simple and straight-forward. But the problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated. In fact, the problem is so complicated that a large number of different theories have been formulated. The principal theories of failure for a member subjected to bi-axial stress are as follows:

1. Maximum principal (or normal) stress theory (also known as Rankine's theory).
2. Maximum shear stress theory (also known as Guest's or Tresca's theory).
3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).
4. Maximum strain energy theory (also known as Haigh's theory).
5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

Since ductile materials usually fail by yielding i.e. when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

1. **Maximum Principal or Normal Stress Theory (Rankine's Theory)**
According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.
Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well-defined yield point) the limiting strength is ultimate stress, therefore according to the above theory, taking factor of safety (F.S.) into consideration, the maximum principal or normal stress (σ_{t1}) in a bi-axial stress system is given by

$$\sigma_{t1} = \sigma_{t1} / F.S., \text{ for ductile materials}$$

$$\sigma_{t1} = \sigma_{u} / F.S., \text{ for brittle materials}$$

Where σ_{yt} = Yield point stress in tension as determined from simple tension test, and
σ_u = Ultimate stress.
Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

2. **Maximum Shear Stress Theory (Guest's or Tresca's Theory)**
According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt} / F.S.$$

Where τ_{max} = Maximum shear stress in a bi-axial stress system,
τ_{yt} = Shear stress at yield point as determined from simple tension test, and
F.S. = Factor of safety.
Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation **(i)** may be written as

$$\tau_{max} = \sigma_{yt} / 2 \times F.S.$$

This theory is mostly used for designing members of ductile materials.

3. **Maximum Principal Strain Theory (Saint Venant's Theory)**
According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (i.e. strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = (\sigma_{t1} / E) - (\sigma_{t2} / m.E)$$

∴ According to the above theory

$$\varepsilon_{max} = (\sigma_{t1} / E) - (\sigma_{t2} / m.E) = \varepsilon = \sigma_{yt} / E \times F.S.$$

Where σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in a bi-axial stress system,
ε = Strain at yield point as determined from simple tension test,
1/m = Poisson's ratio,
E = Young's modulus, and
F.S. = Factor of safety.
From equation **(i),** we may write that

$$\sigma_{t1} - \sigma_{t2} / m = \sigma_{yt} / F.S.$$

This theory is not used, in general, because it only gives reliable results in particular cases.

4. **Maximum Strain Energy Theory (Haigh's Theory)**
According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (i.e. strain energy at the yield point) per unit volume as determined from simple tension test.
We know that strain energy per unit volume in a bi-axial stress system,

$$U_1 = 1/2E\ [(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\ \sigma_{t1}\ X\ \sigma_{t2} / m]$$

And limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = 1/2E\ (\sigma_{yt} / F.S.)^2$$

According to the above theory, U1 = U2

$$1/2E\ [(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\ \sigma_{t1}\ X\ \sigma_{t2} / m] = 1/2E\ (\sigma_{yt} / F.S.)^2$$

$$\text{Or}\ (\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\ \sigma_{t1}\ X\ \sigma_{t2} / m = (\sigma_{yt} / F.S.)^2$$

This theory may be used for ductile materials.

5. **Maximum Distortion Energy Theory (Hencky and Von Mises Theory)**
According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (i.e. distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\ \sigma_{t1}\ X\ \sigma_{t2} = (\sigma_{yt} / F.S.)^2$$

This theory is mostly used for ductile materials in place of maximum strain energy theory.

Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called **stress concentration.**
It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. 1.2. A little consideration will show that the nominal stress in the right and left-hand sides will be uniform but in the region where the cross section is changing, a redistribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

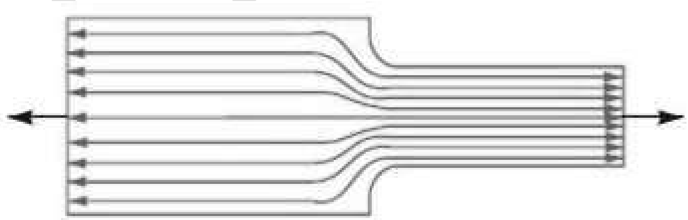


Fig. 1.2 Stress Concentration

Theoretical or Form Stress Concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon the net area.
Mathematically, theoretical or form stress concentration factor,
K_t = Maximum stress / Nominal stress
The value of K_t depends upon the material and geometry of the part.

Stress Concentration due to Holes and Notches

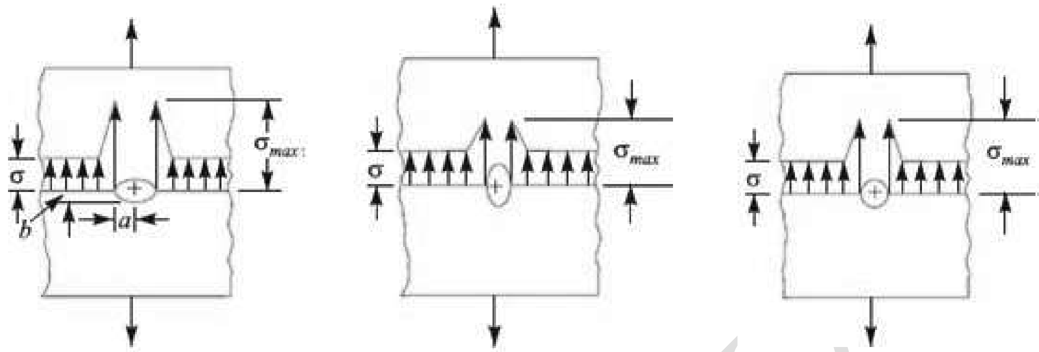
Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig. 1.3. We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\ (\ 1 + 2a/b)$$

And the theoretical stress concentration factor,

$$K_t = \sigma_{max} / \sigma = (1 + 2a/b)$$

When a/b is large, the ellipse approaches a crack transverse to the load and the value of K_t becomes very large. When a/b is small, the ellipse approaches a longitudinal slit [as shown in Fig. 1.3 (b)] and the increase in stress is small. When the hole is circular as shown in Fig. 1.3 (c), then a/b = 1 and the maximum stress is three times the nominal value.



a/b = 2   a/b = 1/2   a/b = 1

$\sigma_{max} = 5\sigma$ $\sigma_{max} = 2\sigma$ $\sigma_{max} = 3\sigma$

(a)   (b)   (c)

Fig. 1.3 Stress concentrations due to holes

The stress concentration in the notched tension member, as shown in Fig. 1.4, is influenced by the depth of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the width of the plate, may be obtained by the following equation,

$$\sigma_{max} = \sigma (1 + 2a/r)$$

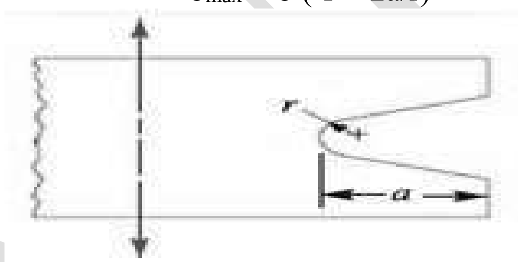


Fig. 1.4 Stress concentrations due to notches

Factors to be considered while designing Machine Parts to Avoid Fatigue Failure

The following factors should be considered while designing machine parts to avoid fatigue failure:

1. The variation in the size of the component should be as gradual as possible.
2. The holes, notches, and other stress raisers should be avoided.
3. The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.
4. The parts should be protected from the corrosive atmosphere.
5. A smooth finish of outer surface of the component increases the fatigue life.
6. The material with high fatigue strength should be selected.
7. The residual compressive stresses over the surface of the part increase its fatigue strength.

Causes of stress concentration

1. The abrupt change in cross-section of machine member e.g. stepped shaft, key way’s, oil groove.
2. The concentrated load applied on small area – examples of this are
i) Contact between wheel & rail.
ii) Contact between ball & race.
iii) Contact between gear teeth.
3. Variation in properties of material From point to point - examples of this are
i) Internal cavities or blowholes.
ii) Cavities in welding.
iii) Nonmetallic inclusions.

Notch sensitivity:
In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term **notch sensitivity** is applied to this behavior. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material.

When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$q = (K_f - 1) / (K_t - 1)$

Or $K_f = 1 + q (K_t - 1)$ [For tensile or bending stress]

And $K_{fs} = 1 + q (K_{ts} - 1)$ [For shear stress]

Where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

Fatigue stress concentration factor:
When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

K_f = Endurance limit without stress concentration / Endurance limit with stress concentration

Factors affecting endurance limit:

1. Load Factor
The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

∴ Endurance limit for reversed bending load

$\sigma_{eb} = \sigma_e \times K_b = \sigma_e$ $(\because K_b = 1)$

Endurance limit for reversed axial load,

$\sigma_{ea} = \sigma_e \times K_a$

And endurance limit for reversed torsional or shear load,

$\tau_e = \sigma_e \times K_s$

2. Surface Finish Factor
When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. Fig. 1.5 shows the values of surface finish factor for the various surface conditions and ultimate tensile strength.

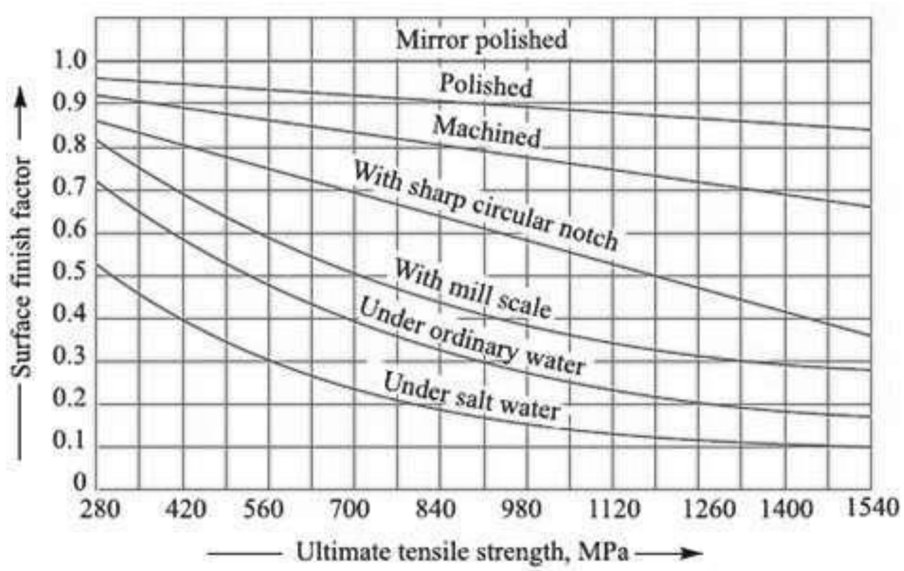


Fig. 1.5 Surface finish factor for various surface conditions

When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finishes factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

Let K_{sur} = Surface finish factor.

∴ Endurance limit,

$\sigma_{e1} = \sigma_{eb} \times K_{sur} = \sigma_e \times K_b \times K_{sur} = \sigma_e \times K_{sur}$ (∵ $K_b = 1$) (For reversed bending load)

$\sigma_{e1} = \sigma_{ea} \times K_{sur} = \sigma_e \times K_a \times K_{sur}$ (For reversed axial load)

$\sigma_{e1} = \tau_e \times K_{sur} = \sigma_e \times K_s \times K_{sur}$ (For reversed torsional or shear load)

3. Size Factor

A little consideration will show that if the size of the standard specimen is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

∴ Endurance limit,

$\sigma_{e2} = \sigma_{e1} \times K_{sz}$ (Considering surface finish factor also)

$\sigma_{e2} = \sigma_{eb} \times K_{sur} \times K_{sz} = \sigma_e \times K_b \times K_{sur} \times K_{sz} = \sigma_e \times K_{sur} \times K_{sz}$ (∵ $Kb = 1$)

$\sigma_{e2} = \sigma_{ea} \times K_{sur} \times K_{sz} = \sigma_e \times K_a \times K_{sur} \times K_{sz}$ (For reversed axial load)

$\sigma_{e2} = \tau_e \times K_{sur} \times K_{sz} = \sigma_e \times K_s \times K_{sur} \times K_{sz}$ (For reversed torsional or shear load)

4. Material Variables

Different materials have different static properties and so also different fatigue properties. Generally, it is seen that the materials which have better static properties are also good under fatigue loading. No exact relationship between fatigue strength and ultimate tensile strength for various materials exists. But for preliminary design, particularly when data are not available, following relations are quite useful.

σ_u = ultimate tensile strength

σ_e = fatigue strength in fully reversed bending load or rotating bending, polished and smooth specimen

σ_{ea} = fatigue strength in fully reversed axial load

τ_e = fatigue strength in fully reversed shearing stress

Then

$\sigma_e = 0.5\ \sigma_u$, for steels

$\sigma_e = 0.4\ \sigma_u$, for cast iron

$\sigma_e = 0.3\ \sigma_u$, for non-ferrous and alloys

$\sigma_{ea} = 0.86\ \sigma_e$

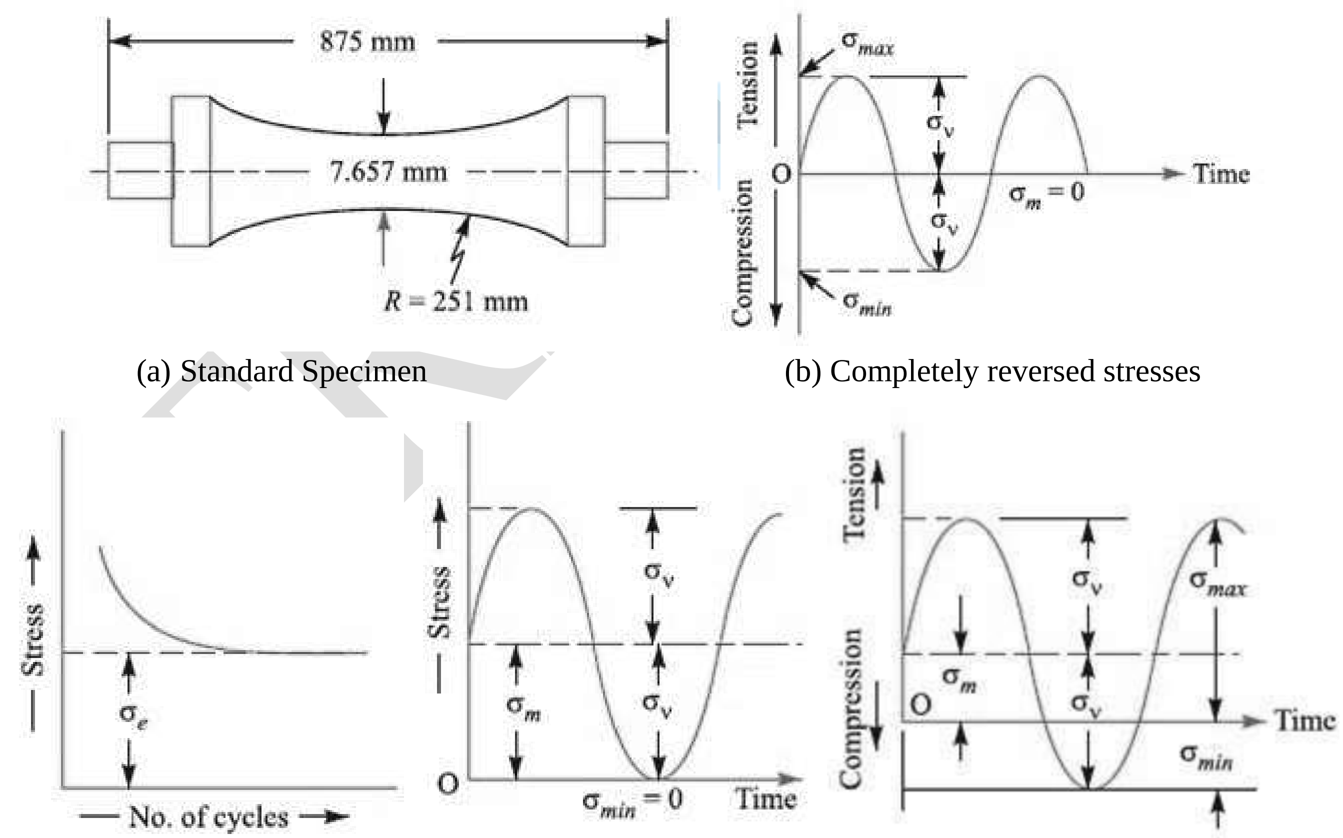
$\tau_e = 0.5\ \sigma_e$, for ductile materials

$\tau_e = 0.2\ \sigma_u$, for non-ferrous metals and alloys
$\tau_e = 0.8\ \sigma_u$, for cast iron

Theoretical stress concentration factor:
The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon the net area.
Mathematically, theoretical or form stress concentration factor,
K_t = Maximum stress / Nominal stress
The value of Kt depends upon the material and geometry of the part.

Cyclic loading/Fatigue
Structures and Machine Elements are subjected repeated loads, called cyclic loads, and the resulting cyclic stresses can lead to microscopic physical damage to the materials involved. Even at stresses well below the material's ultimate strength, this damage can accumulate with continued cycling until it develops into a crack or other damage that leads to failure of the component. The process of accumulating damage and finally to failure due to cyclic loading is called fatigue.

Fatigue and Endurance Limit
It has been found experimentally that when a material is subjected to repeated stresses; it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which is usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is affected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



(a) Standard Specimen (b) Completely reversed stresses

(c) Endurance or Fatigue Limit (d) Repeated Stresses (e) Fluctuating Stresses

Fig. 1.6 Fatigue and Endurance Limit

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 1.6 (a) is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibers varies from

maximum compressive to maximum tensile while the bending stress at the lower fibers varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle.
This is represented by a time-stress diagram as shown in Fig. 1.6 (b). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in a stress-cycle curve as shown in Fig. 1.6 (c). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig. 1.6 (c), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as **endurance** or **fatigue limit** (σ_e). It is defined as the maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for an infinite number of cycles (usually 107 cycles).
It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term **endurance strength** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.
We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 1.6 (b). In actual practice, many machine members undergo a different range of stress than the completely reversed stress.
The stress **versus** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig. 1.6 (e). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v. The following relations are derived from Fig. 1.6 (e):
1. Mean or average stress,
$\sigma_m = (\sigma_{max} + \sigma_{min}) / 2$
2. Reversed stress component or alternating or variable stress,
$\sigma_v = (\sigma_{max} - \sigma_{min}) / 2$

Relation between Endurance Limit and Ultimate Tensile Strength
It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u). Fig. 1.7 shows the endurance limit of steel corresponding to ultimate tensile strength for different surface conditions. Following are some empirical relations commonly used in practice:

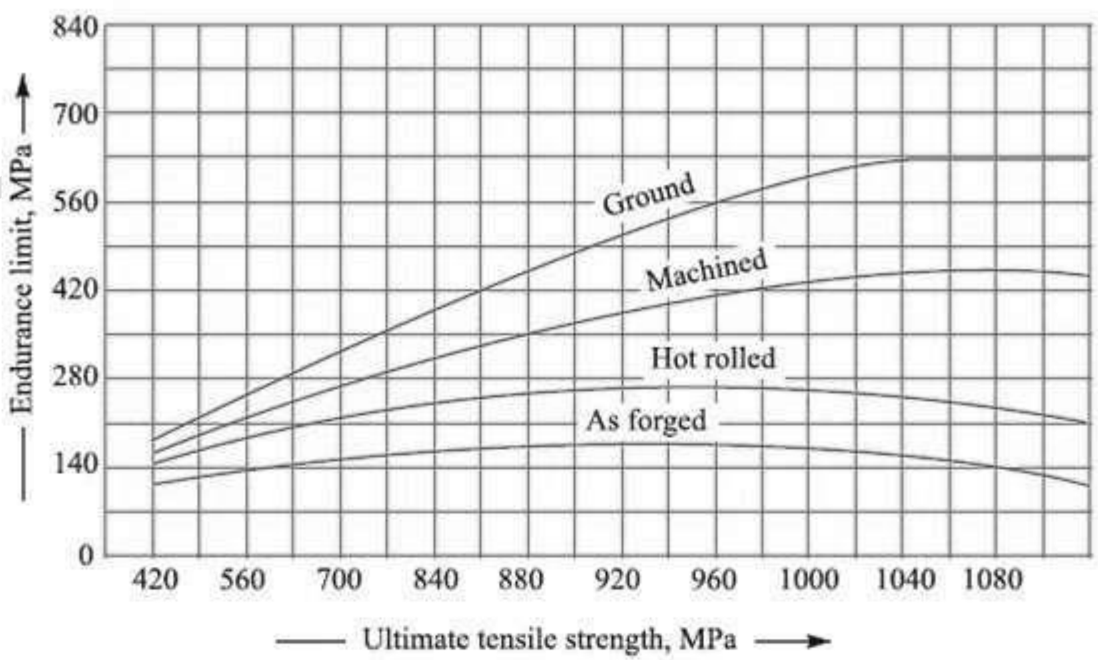


Fig. 1.7 Endurance limit of steel corresponding to ultimate tensile strength

For steel, $\sigma_e = 0.5\ \sigma_u$;
For cast steel, $\sigma_e = 0.4\ \sigma_u$;
For cast iron, $\sigma_e = 0.35\ \sigma_u$;

For non-ferrous metals and alloys, $\sigma e = 0.3\ \sigma u$

Design consideration for fatigue:

Gerber Parabola: -

The relationship between variable stress (σ_v) and mean stress (σ_m) for axial and bending loading for ductile materials are shown in Fig. 1.8. The point σ_e represents the fatigue strength corresponding to the case of complete reversal ($\sigma_m = 0$) and the point σ_u represents the static ultimate strength corresponding to $\sigma_v = 0$. A parabolic curve is drawn between the endurance limit (σ_e) and ultimate tensile strength (σ_u) was proposed by Gerber. Generally, the test data for ductile material fall closer to Gerber parabola as shown in Fig., but because of scattering in the test points, a straight line relationship (i.e. Goodman line and Soderberg line) is usually preferred in designing machine parts.

According to Gerber, variable stress,

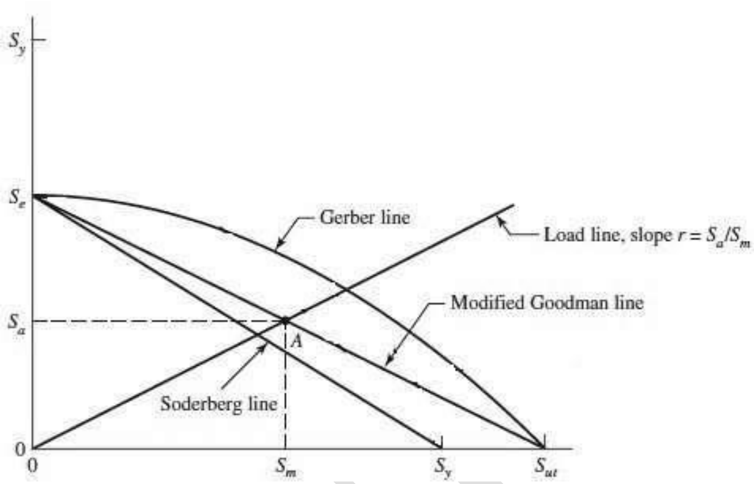


Fig. 1.8 Gerber Method

$\sigma_v = \sigma_e$ [1/F.S. – $(\sigma_m/\sigma_m)^2$ x F.S.]

Where F.S. = Factor of safety,

σ_m = Mean stress (tensile or compressive),

σ_m = Ultimate stress (tensile or compressive), and

σ_e = Endurance limit for reversal loading.

Considering the fatigue stress concentration factor (K_f), the above equation may be written as

1/F.S. = $(\sigma_m/\sigma_m)^2$ x F.S. + (σ_v x K_f)/ σ_e

Goodman Line: -

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σ_u), as shown by line AB in Fig. 1.9, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

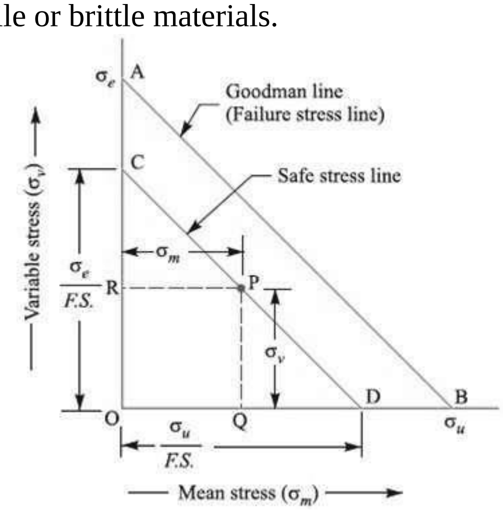


Fig. 1.9 Goodman's Method

In Fig. 1.9, line AB connecting σ_e and σ_u is called **Goodman's failure stress line.** If a suitable factor of safety (F.S.) is applied to endurance limit and ultimate strength, a safe stress line CD may be drawn parallel to the line AB. Let us consider a design point P on the line CD.

Now from similar triangles COD and PQD,

$PQ/CO = QD/OD = (OD – OQ)/OD = 1 – OQ/OD$ $\quad\quad (\because QD = OD – OQ)$

$\sigma_v / \sigma_e/F.S. = 1 - \sigma_m/ \sigma_u/F.S.$

$\sigma_v = \sigma_e/F.S. [1 - \sigma_m/ \sigma_u/F.S.] = \sigma_e [1/F.S. - \sigma_m/ \sigma_u]$

$1/F.S. = \sigma_m/ \sigma_u + \sigma_v/\sigma_e$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads.

Since many machines and structural parts that are subjected to fatigue loads contain regions of high-stress concentration, therefore equation (i) must be altered to include this effect. In such cases, the fatigue stress concentration factor (K_f) is used to multiply the variable stress (σ_v). The above equation may now be written as

$1/F.S. = \sigma_m/ \sigma_u + \sigma_v \text{ x } K_f /\sigma_e$

Where F.S. = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

σ_e = Endurance limit for reversed loading, and

K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the above equation may be written as

$1/F.S. = \sigma_m/ \sigma_u + \sigma_v \text{ x } K_f /\sigma_{eb} \text{ x } K_{sur} \text{ x } K_{sz} = \sigma_m/ \sigma_u + \sigma_v \text{ x } K_f /\sigma_e \text{ x } K_b \text{ x } K_{sur} \text{ x } K_{sz}$

$1/F.S. = \sigma_m/ \sigma_u + \sigma_v \text{ x } K_f /\sigma_e \text{ x } K_{sur} \text{ x } K_{sz}$ $\quad\quad (\because \sigma_e \text{ x } K_b = \sigma_e \text{ and } K_b = 1)$

Where K_b = Load factor for reversed bending load,

K_{sur} = Surface finish factor, and

K_{sz} = Size factor.

Soderberg Line: -

A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line AB in Fig. 1.10, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength.

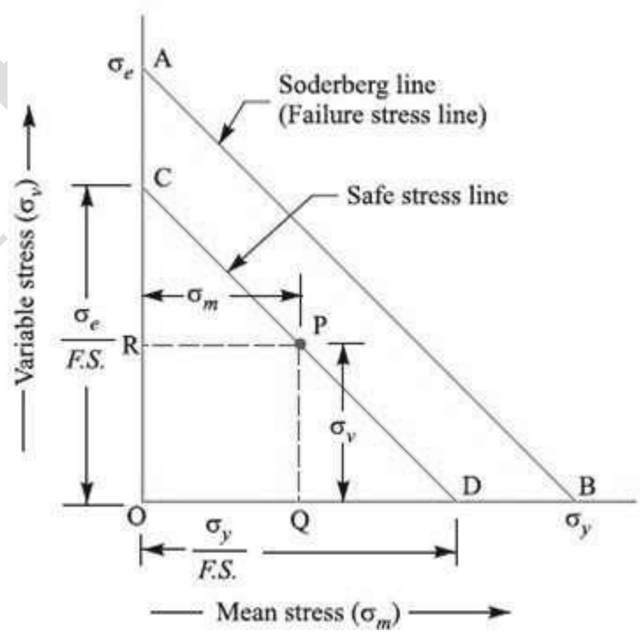


Fig. 1.10 Soderberg Method

The line AB connecting σ_e and σ_y, as shown in Fig. 1.10, is called Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the endurance limit and yield strength, a safe stress line CD may be drawn parallel to the line AB. Let us consider a design point P on the line CD. Now from similar triangles COD and PQD,

PQ/CO = QD/OD = (OD – OQ)/OD = 1 – OQ/OD (∵QD = OD – OQ)

σ_v / σ_e/F.S. = 1 - σ_m / σ_y/F.S.

$\sigma_v = \sigma_e$/F.S. [1 - σ_m / σ_y/F.S.] = σ_e [1/F.S. - σ_m / σ_y]

1/F.S. = $\sigma_m / \sigma_y + \sigma_v / \sigma_e$

For machine parts subjected to fatigue loading, the fatigue stress concentration factor (K_f) should be applied to only variable stress (σ_v). Thus the above equation may be written as

1/F.S. = $\sigma_m / \sigma_y + \sigma_v \times K_f / \sigma_e$

Considering the load factor, surface finish factor and size factor, the above equation may be written as

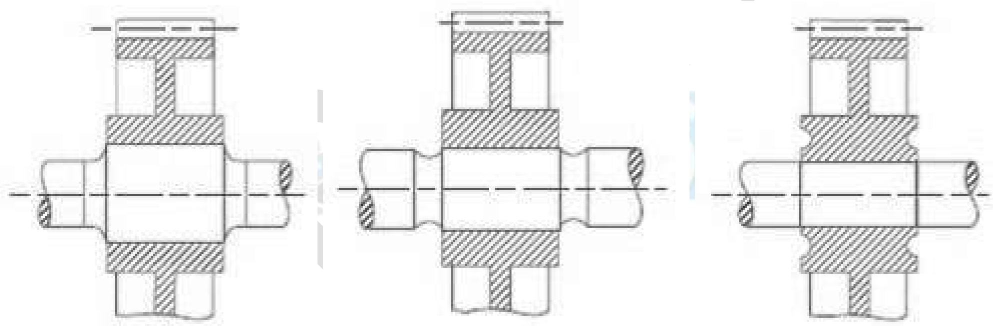
1/F.S. = $\sigma_m / \sigma_u + \sigma_v \times K_f / \sigma_{eb} \times K_{sur} \times K_{sz}$

Since $\sigma_{eb} = \sigma_e \times K_b$ and $K_b = 1$ for reversed bending load, therefore $\sigma_{eb} = \sigma_e$ may be substituted in the above equation.

Factors to be considered while designing machine parts to avoid fatigue failure

The following factors should be considered while designing machine parts to avoid fatigue failure:

1. The variation in the size of the component should be as gradual as possible.
2. The holes, notches, and other stress raisers should be avoided.
3. The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.
4. The parts should be protected from the corrosive atmosphere.
5. A smooth finish of outer surface of the component increases the fatigue life.
6. The material with high fatigue strength should be selected.
7. The residual compressive stresses over the surface of the part increase its fatigue strength.



(a) (b) (c)

Fig. 1.11 Methods of reducing stress concentration of a press fit

Design for finite life

S-N Curve:

To determine the strength of materials under the action of fatigue loads, specimens are subjected to repeated or varying forces of specified magnitudes while the cycles or stress reversals are counted to destruction.

To establish the fatigue strength of a material, quite a number of tests are necessary because of the statistical nature of fatigue. For the rotating-beam test, a constant bending load is applied, and the number of revolutions (stress reversals) of the beam required for failure is recorded. The first test is made at a stress that is somewhat of the ultimate strength of the material. The second test is made at a stress that is less than that used in the first. This process is continued, and the results are plotted as an S-N diagram (Fig. 1.12 a).

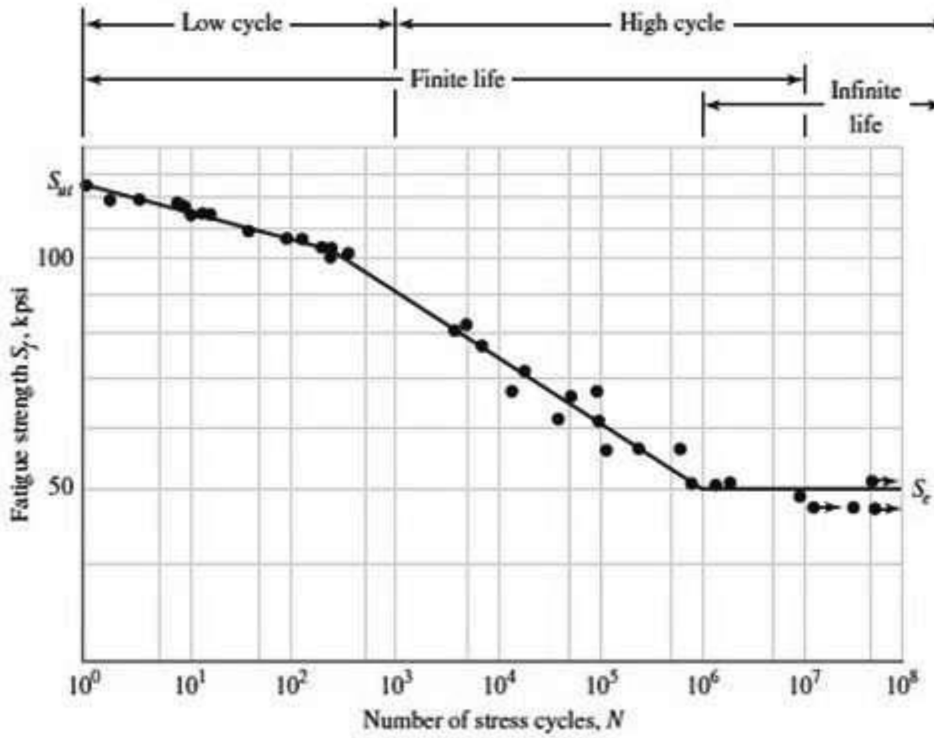


Fig. 1.12 **(a)** An S-N diagram plotted from the results of completely reversed axial fatigue tests

The ordinate of the S-N diagram is called the fatigue strength S_f; a statement of this strength value must always be accompanied by a statement of the number of cycles N to which it corresponds.

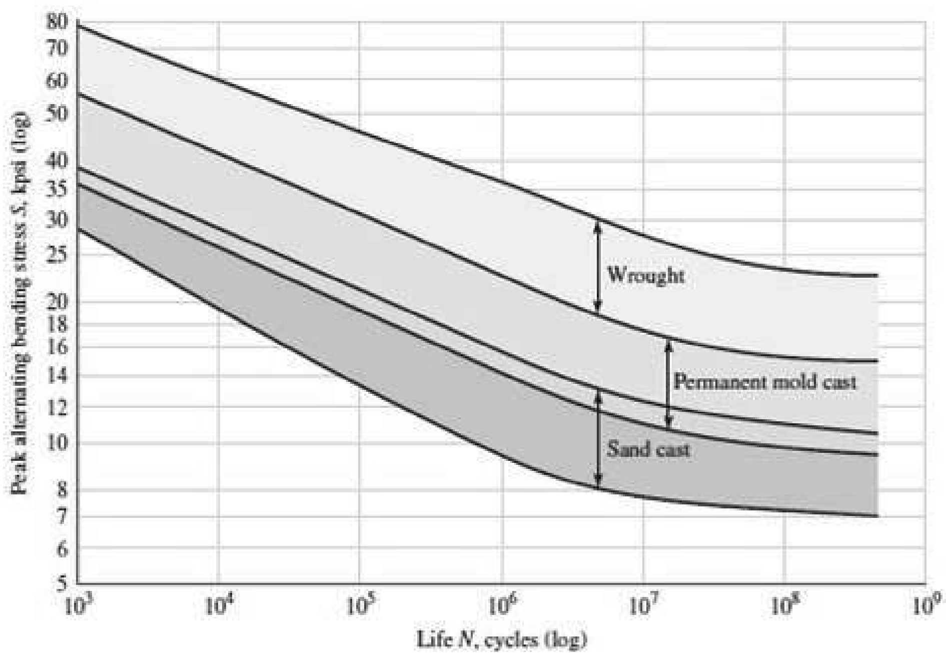


Fig. 1.12 **(b)** S-N bands for representative aluminum alloys, excluding wrought alloys

Soon we shall learn that S-N diagrams can be determined either for a test specimen or for an actual mechanical element. Even when the material of the test specimen and that of the mechanical element are identical, there will be significant differences between the diagrams for the two.

The S-N diagram is usually obtained by completely reversed stress cycles, in which the stress level alternates between equal magnitudes of tension and compression. We note that a stress cycle (N = 1) constitutes a single application and removal of a load and then another application and removal of the load in the opposite direction. Thus N = 1/2 means the load is applied once and then removed, which is the case with the simple tension test.

The body of knowledge available on fatigue failure from N = 1 to N = 1000 cycles is generally classified as low-cycle fatigue, as indicated in Fig. 1.12 (a). High-cycle fatigue, then, is concerned with failure corresponding to stress cycles greater than 10^3 cycles.

We also distinguish a finite-life region and an infinite-life region in Fig. a. The boundary between these regions cannot be clearly defined except for a specific material; but it lies somewhere between 10^6 and 10^7 cycles for steels, as shown in Fig. 1.12 (a).

Because of this necessity for testing, it would really be unnecessary for us to proceed any further in the study of fatigue failure except for one important reason: the desire to know why fatigue failures occur so that the most effective method or methods can be used to improve fatigue strength.

Cumulative fatigue damage

In certain applications, the mechanical component is subjected to different stress levels for different parts of the work cycle. The life of such a component is determined by Miner's equation. Suppose that a component is subjected to completely reversed stresses (σ_1) for (n_1) cycles, (σ_2) for (n_2) cycles, and so on. Let N_1 be the number os stress cycles before fatigue failure if only the alternating stress (σ_1) is acting. One stress cycle will consume ($1/N_1$) of the fatigue life and since there are n1 such cycles at this stress level. The proportionate damage of fatigue life will be [($1/N_1$) n_1) or (n_1/N_1). Similarly, the proportionate damage at stress level (σ_2) will be (n_2/N_2). Adding these quantities, we get,

$(n_1/ N_1) + (n_2/ N_2) + \ldots\ldots. + (n_x/ N_x) = 1$

The above equation is known as Miner's equation. Sometimes, the number of cycles n_1, n_2,…. at stress levels σ_1, σ_2,… etc. Let N be the total life of the component. Then,

$n_1 = \alpha_1 N$

$n_2 = \alpha_2 N$

Substituting these values in Miner's equation,

$(\alpha_1/ N_1) + (\alpha_2/ N_2) + \ldots.. + (\alpha_x/ N_x) = 1/N$

Also,

$\alpha_1 + \alpha_2 + \alpha_3 + \ldots.. + \alpha_x = 1$

With the help of the above equations, the life of the component subjected to different stress levels can be determined.