TRINITY INSTITUTE OF TECHNOLOGY AND RESEARCH BHOPAL

Notes

Theory of Machine (ME-403)

B.TECH

Mechanical Engineering, IV-Semester

ME403 -THEORY OF MACHINES

[1] Introduction, kinematics and kinetics, mechanisms and machines, degree of freedom, types of motions, kinematic concept of links, basic terminology and definitions, joints and kinematic chains, inversions, absolute and relative motions, displacement, velocity and acceleration diagrams, different mechanisms and applications,

[2] kinematic synthesis of linkages, dynamic motion analysis of mechanisms and machines, D'Alembert's principle, number synthesis, free body diagrams, kinematic and dynamic quantities and their relationships, analytical method and graphical method

[3] Cams, introduction, classifications of cams and followers, nomenclature, analysis of cam and follower motion, analytical cam design with specific contours, pressure angle, radius and undercutting, motion constrains and program, critical path motion, torque on cam shaft

[4] Power transmission, kinematics of belt- pulley, flat and v –belt, rope, condition of maximum power transmission, efficiency, friction, friction devices, pivot and collars, power screw, plate and cone clutch, brakes, classifications, bock, band, internal and external, friction circle, friction axis,

[5] Gears, laws of gearing, classification and basic terminology, tooth profiles, kinematic considerations, types of gears, spur, bevel, worm, helical, hypoid etc, gear trains, epicyclic, compound,, balancing- static and dynamic, in same/ different planes, Introduction to vibration, single degree of freedom.

BOOKS:

[1] R.L.Norton,kinematics& dynamics of machinery,Tata McGraw Hill, ISBN13 978 0 07 014480 4
[2] A.Ghosh & A.Malik, Theory of Mechanisms and Machines,EWP Pvt Ltd,ISB 81 85095 72 8

Unit 1 -Introduction of mechanisms

1.1 Introduction

Mechanics: It is that branch of scientific analysis which deals with motion, time and force.

Kinematics is the study of motion, without considering the forces which produce that motion. Kinematics of machines deals with the study of the relative motion of machine parts. It involves the study of position, displacement, velocity and acceleration of machine parts.

Dynamics of machines involves the study of forces acting on the machine parts and the motions resulting from these forces.

Plane motion: A body has plane motion, if all its points move in planes which are parallel to some reference plane. A body with plane motion will have only three degrees of freedom. I.e., linear along two axes parallel to the reference plane and rotational/angular about the axis perpendicular to the reference plane. (eg. linear along X and Z and rotational about Y.)The reference plane is called plane of motion. Plane motion can be of three types. 1) Translation 2) rotation and 3) combination of translation and rotation.

Kinematic link (or) element

A machine part or a component of a mechanism is called a kinematic link or simply a link. A link is assumed to be completely rigid, or under the action of forces it does not suffer any deformation, signifying that the distance between any two points on it remains constant. Although all real machine parts are flexible to some degree, it is common practice to assume that deflections are negligible and parts are rigid when analyzing a machine's kinematic performance.

Types of link

(a) Based on number of elements of link:

Binary link: Link which is connected to other links at two points. (Fig.1.3 a) **Ternary link:** Link which is connected to other links at three points. (Fig.1.3 b) **Quaternary link:** Link which is connected to other links at four points. (Fig1.3 c)

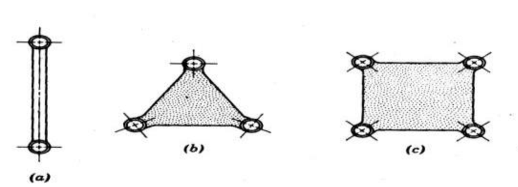


Fig.1.3

(a) Based on type of structural behavior:

Sometimes, a machine member may possess one-way rigidity and is capable of transmitting the force in one direction with negligible deformation. Examples are (a) chains, belts and ropes which are resistant to tensile forces, and (b) fluids which are resistant to compressive forces and are used as links in hydraulic presses, brakes and jacks. In order to transmit motion, the driver and the follower may be connected by the following three types of links:

1. ***Rigid link.*** A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

2. ***Flexible link.*** A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. ***Fluid link.*** A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

1.4 Structure

It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure.

Machine: A machine is a mechanism or collection of mechanisms, which transmit force from the source of power to the resistance to be overcome. Though all machines are mechanisms, all mechanisms are not machines. Many instruments are mechanisms but are not machines, because they do no useful work nor do they transform energy.

Difference between structure & machine

The following differences between a machine and a structure are important from the subject point of view:

1. The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another.

2. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work.

3. The links of a machine may transmit both power and motion, while the members of a structure transmits forces only.

Comparison of Mechanism, Machine and Structure

Mechanism	Machine	Structure
1. There is relative motion between the parts of a mechanism	Relative motion exists between parts of a machine.	There is no relative motion between the members of a structure. It is rigid as a whole.
2. A mechanism modifies and transmits motion.	A machine consists of one or more mechanisms and hence transforms motion	A structure does not transform motion.
3. A mechanism does not transmit forces and does not do work	A machine modifies energy or do some work	A structure does not do work. It only transmits forces.
4. Mechanisms are dealt with in kinematics.	Machines are dealt with in kinetics.	Structures are dealt with in statics.

1.6 Kinematic pair

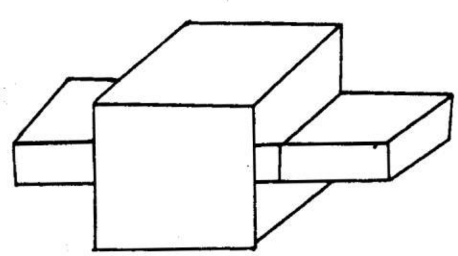
The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as **kinematic pair**.

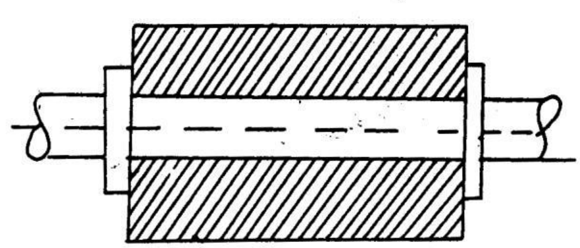
Classification of kinematic pair

The kinematic pairs may be classified according to the following considerations :

(i) Based on nature of contact between elements:

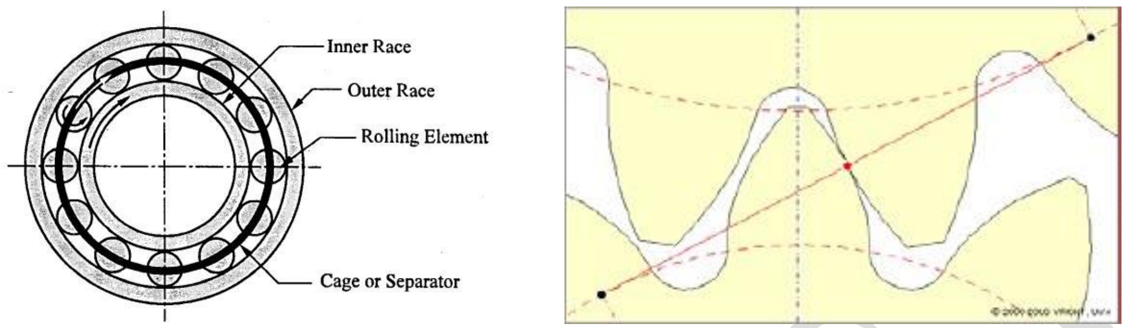
(a) Lower pair. If the joint by which two members are connected has surface contact, the pair is known as lower pair. Eg. pin joints, shaft rotating in bush, slider in slider crank mechanism.





Lower pairs

(b) Higher pair. If the contact between the pairing elements takes place at a point or along a line, such as in a ball bearing or between two gear teeth in contact, it is known as a higher pair.



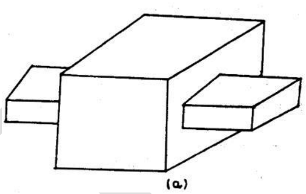
Higher pairs

(ii) Based on relative motion between pairing elements:

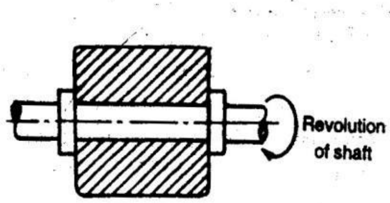
(a) Siding pair. Sliding pair is constituted by two elements so connected that one is constrained to have a sliding motion relative to the other. DOF = 1

(b) Turning pair (revolute pair). When connections of the two elements are such that only a constrained motion of rotation of one element with respect to the other is possible, the pair constitutes a turning pair. DOF = 1

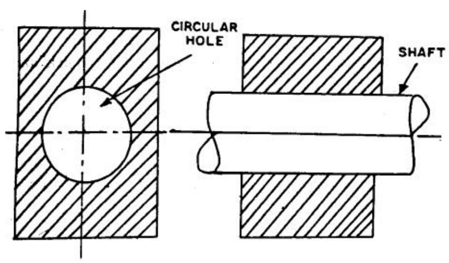
(c) Cylindrical pair. If the relative motion between the pairing elements is the combination of turning and sliding, then it is called as cylindrical pair. DOF = 2



Sliding pair

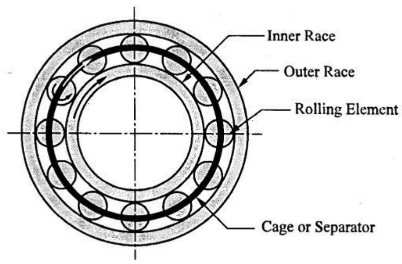


Turning pair

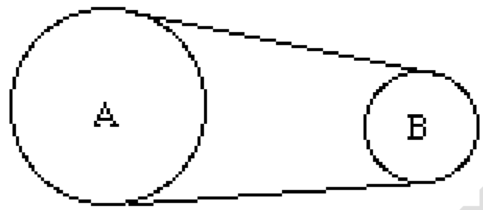


Cylindrical pair

(d) Rolling pair. When the pairing elements have rolling contact, the pair formed is called rolling pair. Eg. Bearings, Belt and pulley. DOF = 1



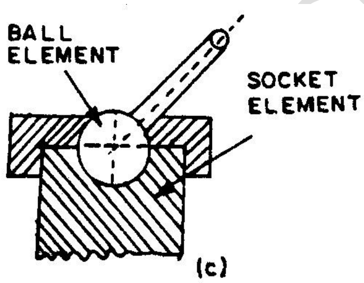
Ball bearing



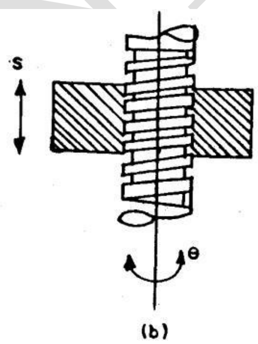
Belt and pulley

(e) Spherical pair. A spherical pair will have surface contact and three degrees of freedom. Eg. Ball and socket joint. DOF = 3

(f) Helical pair or screw pair. When the nature of contact between the elements of a pair is such that one element can turn about the other by screw threads, it is known as screw pair. Eg. Nut and bolt. DOF = 1



Ball and socket joint



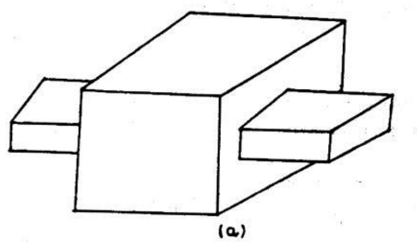
Screw pair

(a) Sliding pair (prismatic pair) eg. piston and cylinder, crosshead and slides, tail stock on lathe bed.
(b) Turning pair (Revolute pair): eg: cycle wheel on axle, lathe spindle in head stock.
(c) Cylindrical pair: eg. shaft turning in journal bearing.
(d) Screw pair (Helical pair): eg. bolt and nut, lead screw of lathe with nut, screw jack.
(e) Spherical pair: eg. penholder on stand, castor balls.

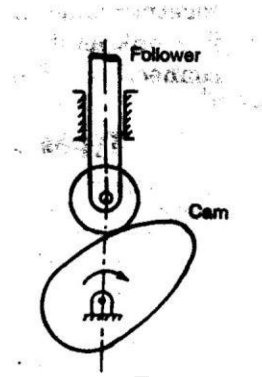
(iii) Based on the nature of mechanical constraint.

(a) **Closed pair.** Elements of pairs held together mechanically due to their geometry constitute a closed pair. They are also called form-closed or self-closed pair.

(b) **Unclosed or force closed pair.** Elements of pairs held together by the action of external forces constitute unclosed or force closed pair .Eg. Cam and follower.



Closed pair

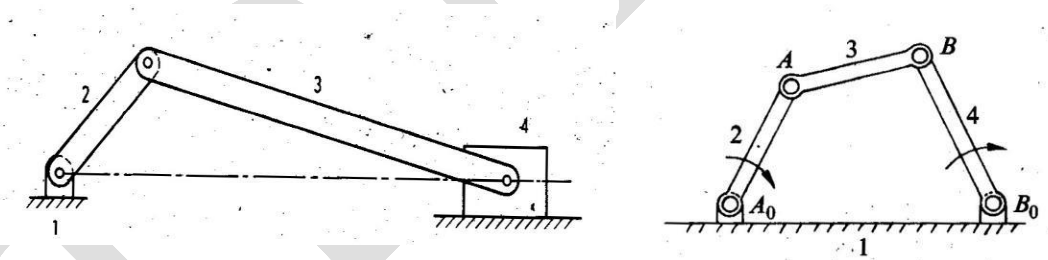


Force closed pair (cam & follower)

Mechanism

When one of the links of a kinematic chain is fixed, the chain is known as ***mechanism***. A mechanism with four links is known as ***simple mechanism***, and the mechanism with more than four links is known as ***compound mechanism***. When a mechanism is required to transmit power or to do some particular type of work, it then becomes a ***machine***.

A mechanism is a constrained kinematic chain. This means that the motion of any one link in the kinematic chain will give a definite and predictable motion relative to each of the others. Usually one of the links of the kinematic chain is fixed in a mechanism.

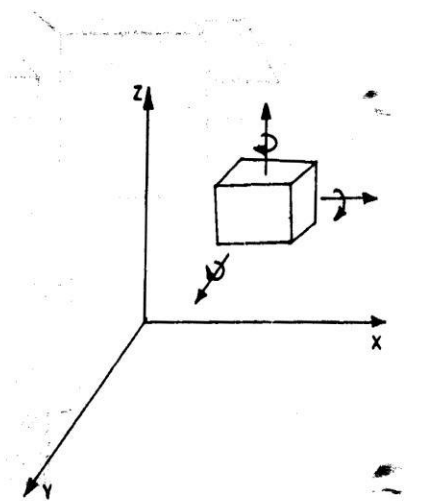


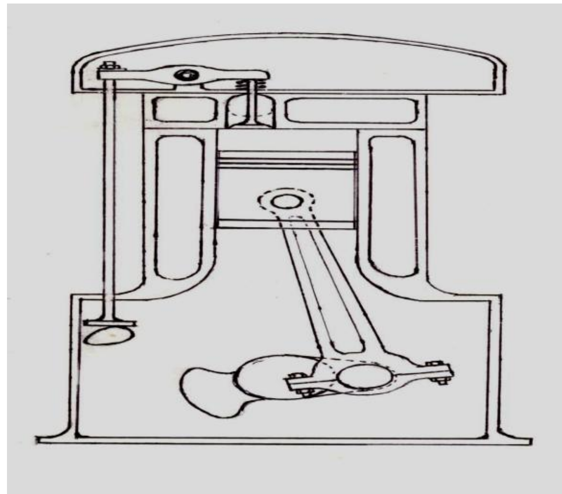
Slider crank and four bar mechanisms.

Number of degrees of freedom for plane mechanism

Degrees of freedom/mobility of a mechanism: The number of independent input parameters (or pair variables) that are needed to determine the position of all the links of the mechanism with respect to the fixed link is termed its degrees of freedom.

Degrees of freedom (DOF) is the number of independent coordinates required to describe the position of a body in space. A free body in space can have six degrees of freedom. I.e., linear positions along x, y and z axes and rotational/angular positions with respect to x, y and z axes. In a kinematic pair, depending on the constraints imposed on the motion, the links may loose some of the six degrees of freedom.





I.C.Engine

Planar mechanisms: When all the links of a mechanism have plane motion, it is called as a planar mechanism. All the links in a planar mechanism move in planes parallel to the reference plane.

Serial Mechanisms (Manipulators): Early manipulators were work holding devices in manufacturing operations so that the work piece could be manipulated or brought to different orientations with respect to the tool head. Welding robots of the auto industry and assembly robots of IC manufacture are examples.

Application of kutzbach criterion to Plane mechanisms

$$\boxed{F = 3(n-1) - 2l - h}$$

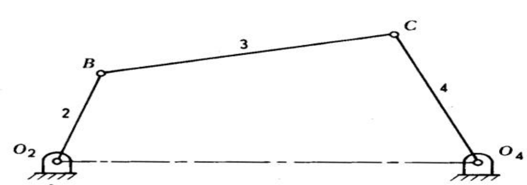
Where n=number of links; l= number of lower joints (or) pairs and h= number of higher pairs (or) joints

This is called the Kutzbach criterion for the mobility of a planar mechanism.

Inversion of Mechanism

A mechanism is one in which one of the links of a kinematic chain is fixed. Different mechanisms can be obtained by fixing different links of the same kinematic chain. These are called as inversions of the mechanism.

Inversions of Four Bar Chain



One of the most useful and most common mechanisms is the four-bar linkage. In this mechanism, the link which can make complete rotation is known as crank (link 2). The link which oscillates is known as rocker or lever (link 4). And the link connecting these two is known as coupler (link 3). Link 1 is the frame.

Inversions:

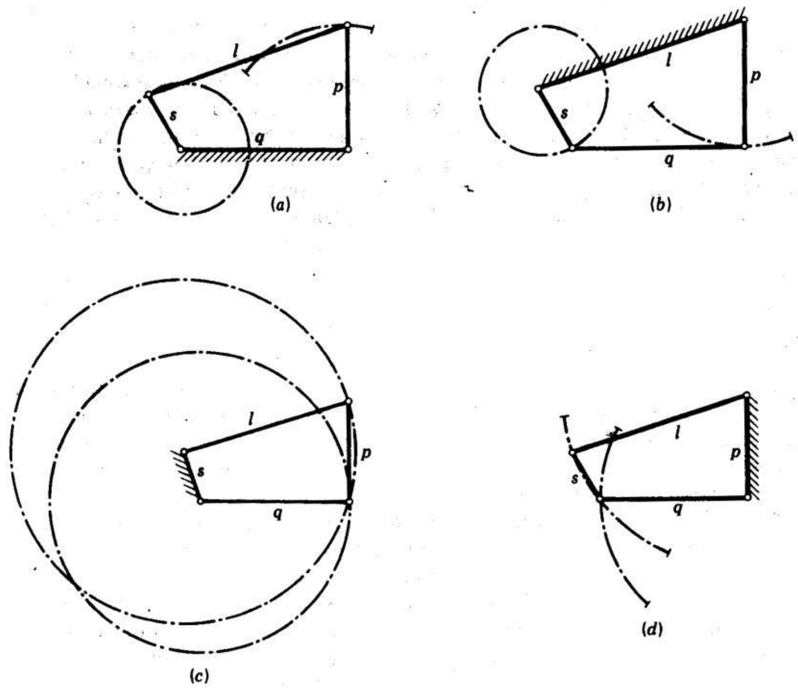
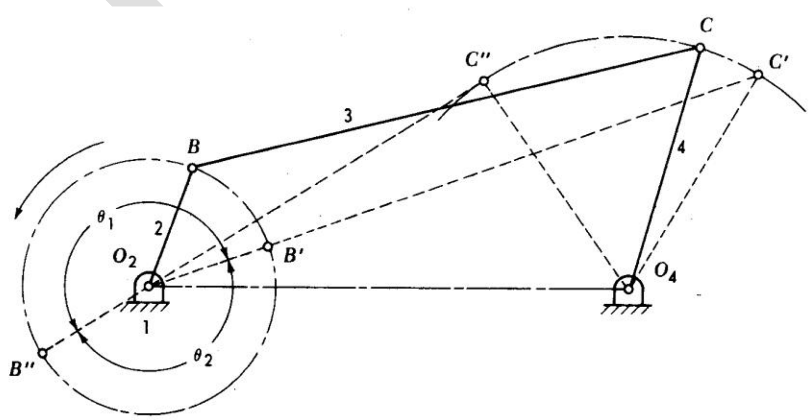


Fig.1.23 Inversions of four bar chain.

Crank-rocker mechanism: In this mechanism, either link 1 or link 3 is fixed. Link 2 (crank) rotates completely and link 4 (rocker) oscillates. It is similar to (a) or (b) of fig.1.23.



Double crank mechanism (Coupling rod of locomotive).

This is one type of drag link mechanism, where, links 1& 3 are equal and parallel and links 2 & 4 are equal and parallel.

The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links in the fig. in this mechanism, th e links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

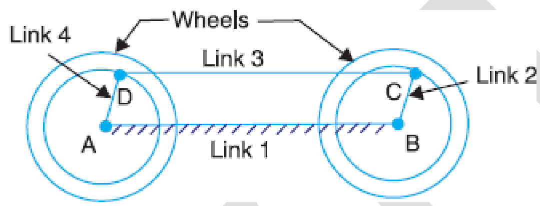
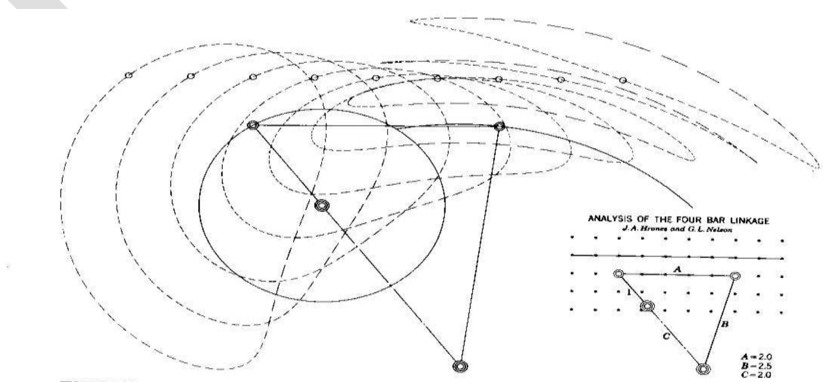


Fig.1.25

Double rocker mechanism. In this mechanism, link 4 is fixed. Link 2 makes complete rotation, whereas links 3 & 4 oscillate (Fig.1.23d)

Coupler Curves: The link connecting the driving crank with the follower crank in a four bar linkage is called the coupler. Similarly, in the case of a single slider crank mechanism the connecting rod is the coupler. During the motion of the mechanism any point attached to the coupler generates some path with respect to the fixed link. This path is called the coupler curve. The point, which generates the path is variously called the coupler point, trace point, tracing point, or tracer point.

An example of the coupler curves generated by different coupler points is given in figure below. Mechanisms can be designed to generate any curve.



Inversions of Single Slider Chain

Slider crank chain: This is a kinematic chain having four links. It has one sliding pair and three turning pairs. Link 2 has rotary motion and is called crank. Link 3 has got combined rotary and reciprocating motion and is called connecting rod. Link 4 has reciprocating motion and is called slider. Link 1 is frame (fixed). This mechanism is used to convert rotary motion to reciprocating and vice versa.

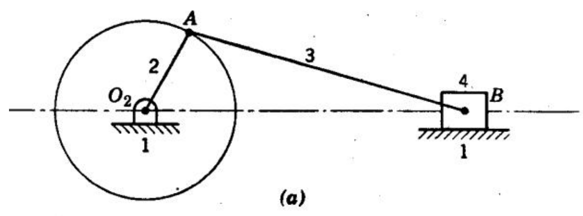
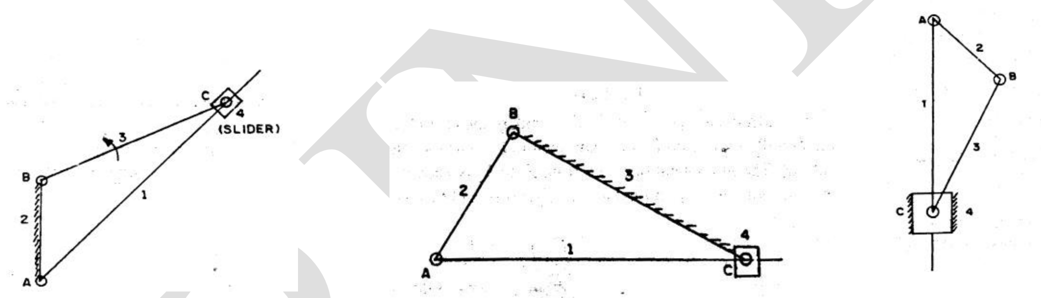


Fig1.27

Inversions of slider crank chain

Inversions of slider crank mechanism is obtained by fixing links 2, 3 and 4.



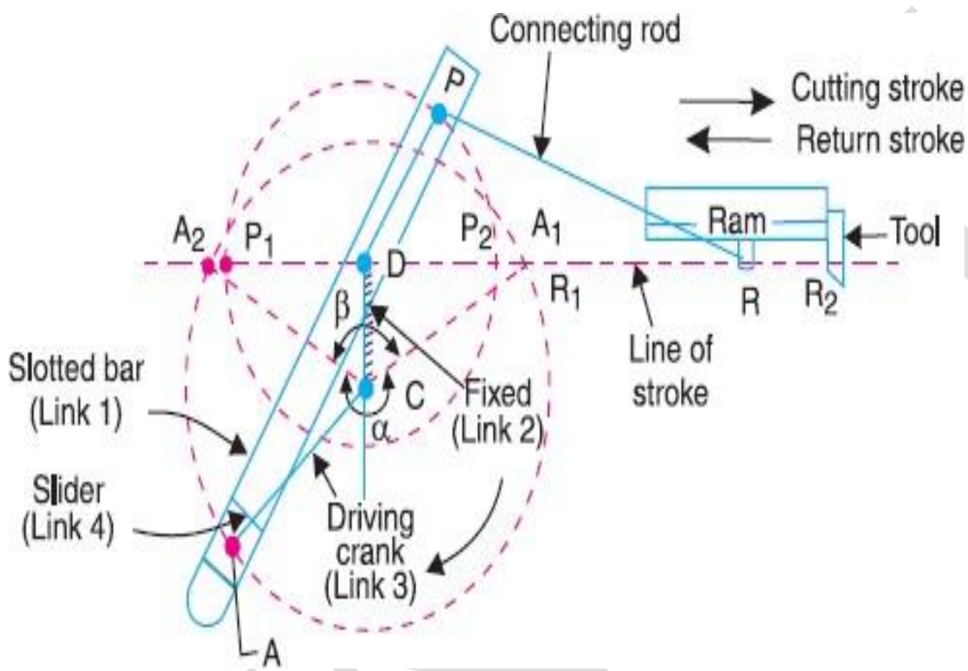
(a) crank fixed (b) connecting rod fixed (c) slider fixed

Fig.1.28

Quick return motion mechanisms.

Quick return mechanisms are used in machine tools such as shapers and power driven saws for the purpose of giving the reciprocating cutting tool a slow cutting stroke and a quick return stroke with a constant angular velocity of the driving crank.

Whitworth quick return motion mechanism–Inversion of slider crank mechanism. This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link *CD* (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank *CA* (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, *i.e.* along a line passing through D and perpendicular to CD.



When the driving crank CA moves from the position $CA1$ to $CA2$ (or the link DP from the position $DP1$ to $DP2$) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2 PD.

Now when the driving crank moves from the position $CA2$ to $CA1$ (or the link DP from $DP2$ to $DP1$) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (*i.e.* during forward or cutting stroke) will be equal to the time taken by the driving crank to move from $CA1$ to $CA2$. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from $CA2$ to $CA1$.

Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke.

The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha}$$

Crank and slotted lever quick return motion mechanism – Inversion of slider crank mechanism (connecting rod fixed).

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines. In this mechanism, the link *AC* (*i.e.* link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank *CB* revolves with uniform angular speed about the fixed centre *C*. A sliding block attached to the crank pin at *B* slides along the slotted bar *AP* and thus causes *AP* to oscillate about the pivoted point *A*. A short link *PR* transmits the motion from *AP* to the ram which carries the tool

of stroke $R1R2$. The line of stroke of the ram (*i.e.* $R1R2$) is perpendicular to *AC* produced.

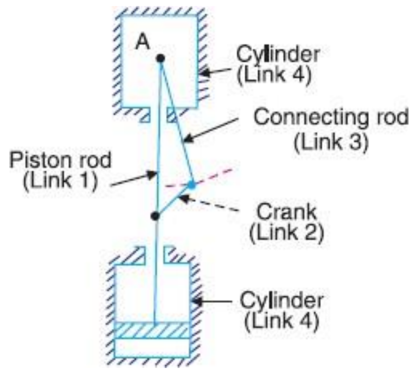
We see that the angle β made by the forward or cutting stroke is greater than the angle α described by the return stroke. Since the crank rotates with uniform angular speed, therefore the return stroke is completed within shorter time. Thus it is called quick return motion mechanism.

In the extreme positions, $AP1$ and $AP2$ are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position $CB1$ to $CB2$ (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position $CB2$ to $CB1$ (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta}$$

Pendulum pump or bull engine–Inversion of slider crank mechanism (slider fixed). In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (*i.e.* sliding pair), as shown in Fig. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at *A* and the piston

piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig.



VELOCITY AND ACCELERATION ANALYSIS OF MECHANISMS

In this, we shall discuss the relative velocity method for determining the velocity of different points in the mechanism. The study of velocity analysis is very important for determining the acceleration of points in the mechanisms.

Kinematics deals with study of relative motion between the various parts of the machines. Kinematics does not involve study of forces. Thus motion leads study of displacement, velocity and acceleration of a part of the machine. As dynamic forces are a function of acceleration and acceleration is a function of velocities, study of velocity and acceleration will be useful in the design of mechanism of a machine. The mechanism will be represented by a line diagram which is known as configuration diagram. The analysis can be carried out both by graphical method as well as analytical method.

Displacement: All particles of a body move in parallel planes and travel by same distance is known, linear displacement and is denoted by 'x'. A body rotating about a fired point in such a way that all particular move in circular path angular displacement and is denoted by '□'.

Velocity: Rate of change of displacement is velocity. Velocity can be linear velocity of angular velocity.

Unit:II KINEMATIC SYNTHESIS OF LINKAGES

Kinematic synthesis, also known as *mechanism synthesis*, determines the size and configuration of mechanisms that shape the flow of power through a mechanical system, or machine, to achieve a desired performance. The word *synthesis* refers to combining parts to form a whole. Hartenberg and Denavit describe kinematic synthesis as

...it is design, the creation of something new. Kinematically, it is the conversion of a motion idea into hardware.

The earliest machines were designed to amplify human and animal effort, later gear trains and linkage systems captured wind and flowing water to rotate millstones and pumps. Now machines use chemical and electric power to manufacture, transport, and process items of all types. And kinematic synthesis is the collection of techniques for designing those elements of these machines that achieve required output forces and movement for a given input.

Linkages are the basic building blocks of all mechanisms. All common forms of mechanisms (cams, gears, belts, chains) are in fact variations on a common theme of linkages. Linkages are made up of links and joints.

- **Links**: rigid member having nodes.
- **Node**: attachment points.
- **Binary link**: 2 nodes
- **Ternary link**: 3 nodes
- **Quaternary link**: 4 nodes
- **Joint**: connection between two or more links (at their nodes) which allows motion; (Joints also called kinematic pairs)

Joints can be classified in several ways:

1. By the type of contact between the elements, line, point, or surface.
2. By the number of degrees of freedom allowed at the joint.
3. By the type of physical closure of the joint: either force or form closed.
4. By the number of links joined (order of the joint).

A more useful means to classify joints (pairs) is by the number of degrees of freedom that they allow between the two elements joined. A joint with more than one freedom may also be a higher pair.

- Joint order = number of links-1

Dynamic Force Analysis

- ***D'Alembert's Principle and Inertia Forces:*** An important principle, known as d'Alembert's principle, can be derived from Newton's second law. In words, d'Alembert's principle states that the reverse-effective forces and torques and the external forces and torques on a body together give statical equilibrium.

$F + (-ma_G) = 0$

$T_{eG} + (-I_G\alpha) = 0$

- The terms in parentheses in above equations are called the reverse-effective force and the reverse-effective torque, respectively. These quantities are also referred to as inertia force and inertia torque. Thus, we define the inertia force ***F***, as

$\boldsymbol{F_i = -ma_G}$

- This reflects the fact that a body resists any change in its velocity by an inertia force proportional to the mass of the body and its acceleration. The inertia force acts through the center of mass ***G*** of the body. The inertia torque or inertia couple ***C***, is given by:

$C_i = -I_G\alpha$

As indicated, the inertia torque is a pure torque or couple.

$$\sum F = \sum F_e + F_i = 0$$
$$\sum T_G = \sum T_{eG} + C_i = 0$$

- Where $\sum F$ refers here to the summation of external forces and, therefore, is the resultant external force, and $\sum T_{eG}$ is the summation of external moments, or resultant external moment, about the center of mass ***G***.
- Thus, the dynamic analysis problem is reduced in form to a static force and moment balance where inertia effects are treated in the same manner as external forces and torques.
- In particular for the case of assumed mechanism motion, the inertia forces and couples can be determined completely and thereafter treated as known mechanism loads.
- Furthermore, D'Alembert's principle facilitates moment summation about any arbitrary point ***P*** in the body, if we remember that the moment due to inertia force ***F*,** must be included in the summation. Hence,

$\sum T_P = \sum T_{eP} + C_i + R_{PG} \times F_t = 0$

- Where $\sum T_P$ is the summation of moments, including inertia moments, about point ***P***. $\sum T_{eP}$ is the summation of external moments about ***P, C***, is the inertia couple, is the inertia force, and $\boldsymbol{R_{PG}}$ is a vector from point P to point ***C***.

For a body in plane motion in the ***xy*** plane with all external forces in that plane.

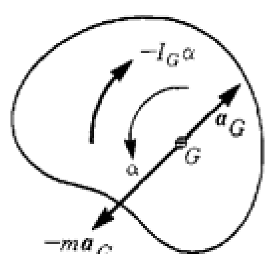
$$\sum F_x = \sum F_{ex} + F_{ix} = \sum F_{ex} + (-m a_{Gx}) = 0$$
$$\sum F_y = \sum F_{ey} + F_{iy} = \sum F_{ey} + (-m a_{Gy}) = 0$$
$$\sum T_G = \sum T_{eG} + C_i = \sum T_{eG} + (-I_G \alpha) = 0$$

- Where $\boldsymbol{a_{Gx}}$ and $\boldsymbol{a_{Gy}}$ are the ***x*** and ***y*** components of $\boldsymbol{a_G}$. These are three scalar equations, where the sign convention for torques and angular accelerations is based on a right-hand ***xyz*** coordinate system; that is. Counterclockwise is positive and clockwise is negative. The general moment summation about arbitrary point ***P***,

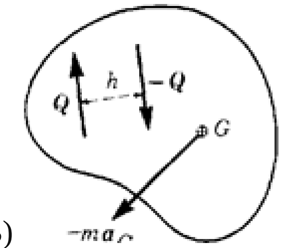
$$\sum T_P = \sum T_{eP} + C_i + R_{PGx} . F_{iy} - R_{PGy} . F_{ix}$$
$$= \sum T_{eP} + (-I_G \alpha) + R_{PGx}(-m a_{Gy}) - R_{PGy}(-m a_{Gx}) = 0$$

Where $\boldsymbol{R_{PGx}}$ and $\boldsymbol{R_{PGy}}$ are the ***x*** and ***y*** components of position vector $\boldsymbol{R_{PG}}$. This expression for dynamic moment equilibrium will be useful in the analyses to be presented in the following sections of this chapter.

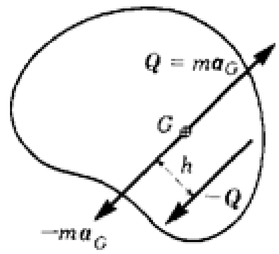
- ***Equivalent Offset Inertia Force:*** For purposes of graphical plane force analysis, it is convenient to define what is known as the equivalent offset inertia force. This is a single force that accounts for both translational inertia and rotational inertia corresponding to the plane motion of a rigid body.
  - Figure A shows a rigid body with planar motion represented by center of mass acceleration $\boldsymbol{a_C}$ and angular acceleration α. The inertia force and inertia torque associated with this motion are also shown. The inertia torque $-I_G\alpha$ can be expressed as a couple consisting of forces ***Q*** and (- ***Q***) separated by perpendicular



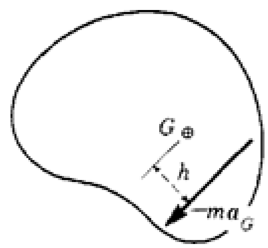
(A)



(B)



(C)



(D)

- Figure (A) Derivation of the equivalent offset inertia force associated with planer motion of a rigid body.
- Figure(B) Replacement of the inertia torque by a couple.
- Figure(C) The strategic choice of a couple.
- Figure(D) The single force is equivalent to the combination of a force and a torque in figure (A)

$$h = \left|I_G \alpha\right| / \left|m a_G\right|$$

Distance ***h***, as shown in Figure B. The necessary conditions for the couple to be equivalent to the inertia torque are that the sense and magnitude be the same. Therefore, in this case, the sense of the couple must be clockwise and the magnitudes of ***Q*** and ***h*** must satisfy the relationship

$$\left|Q.h\right| = \left|I_G.\alpha\right|$$

Otherwise, the couple is arbitrary and there are an infinite number of possibilities that will work. Furthermore, the couple can be placed anywhere in the plane.

Figure C shows a special case of the couple, where force vector ***Q*** is equal to $\boldsymbol{m a_G}$ and acts through the center of mass. Force ***(- Q)*** must then be placed as shown to produce a clockwise sense and at a distance;

$$h = \frac{|I_G\alpha|}{|Q|} = \frac{|I_G\alpha|}{|ma_G|}$$

Force $\boldsymbol{Q}$ will cancel with the inertia force $\boldsymbol{F_i = -\, ma_G}$, leaving the single equivalent offset force, which has the following characteristics:

1. The magnitude of the force is $|\, \boldsymbol{ma_G} \,|$.
2. The direction of the force is opposite to that of acceleration α.
3. The perpendicular offset distance from the center of mass to the line of action of the force .
4. The force is offset from the center of mass so as to produce a moment about the center of mass that is opposite in sense to acceleration $\boldsymbol{a}$.

The usefulness of this approach for graphical force analysis will be demonstrated in the following section. It should be emphasized, however, that this approach is usually unnecessary in analytical solutions.. Including the original inertia force and inertia torque, can be applied directly.

- ***Dynamic Analysis of the Four-Bar Linkage:***
    - The analysis of a four-bar linkage will effectively illustrate most of the ideas that have been presented; furthermore, the extension to other mechanism types should become clear from the analysis of this mechanism.
- ***Dynamic Analysis of the Slider-Crank Mechanism:***
    - Dynamic forces are a very important consideration in the design of slider crank mechanisms for use in machines such as internal combustion engines and reciprocating compressors.
    - Following such a process a kinematics analysis is first performed from which expressions are developed for the inertia force and inertia torque for each of the moving members, These quantities may then be converted to equivalent offset inertia forces for graphical analysis or they may be retained in the form of forces and torques for analytical solution.
    - Following figure is a schematic diagram of a slider crank mechanism, showing the crank 1, the connecting rod 2, and the piston 3, all of which are assumed to be rigid. The center of mass locations are designated by letter **G,** and the members have masses $\boldsymbol{m}$, and moments of inertia $\boldsymbol{I_{Gi}}$, i = 1, 2, 3.
    - The following analysis will consider the relationships of the inertia forces and torques to the bearing reactions and the drive torque on the crank, at an arbitrary mechanism position given by crank angle φ Friction will be neglected.

Free Body Diagram

The free body diagram helps you understand and solve static and dynamic problem involving forces. It is a diagram including all forces acting on a given object without the other object in the system. You need to first understand all the forces acting on the object and then represent these force by arrows in the direction of the force to be drawn.

Examples of Free Body Diagrams with Detailed Explanations

Example 1 : A book on a table

In this example, there are two forces acting on a book at rest on a table:

1) The weight W exerted by the earth on the book
2) The normal force N exerted by the table on the book.

Example 2 : A suspended block

In this example, there are two forces acting on the suspended block at rest:

1) The weight W exerted by the earth on the block
2) The tension force T exerted by the string (or rope) on the block

Example 3 : A block on a floor with an acting force Fa to pull the block

In this example, the block is being pulled and therefore a force of friction acts on the block. So there are four forces acting on the block.

1) The weight W exerted by the earth on the block
2) The normal force N exerted by floor on the block.
3) The acting force Fa to pull the block.
4) The force of friction F_f exerted by floor on the block in the direction opposite the motion

Unit:III CAM

Cam - A mechanical device used to transmit motion to a follower by direct contact. Where Cam – driver member

Follower - driven member.

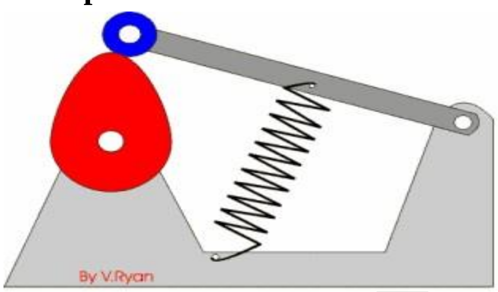
The cam and the follower have line contact and constitute a higher pair.

In a cam - follower pair, the cam normally rotates at uniform speed by a shaft, while the follower may is predetermined, will translate or oscillate according to the shape of the cam. A familiar example is the camshaft of an automobile engine, where the cams drive the push rods (the followers) to open and close the valves in synchronization with the motion of the pistons.

Applications:

The cams are widely used for operating the inlet and exhaust valves of Internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes.

Example of cam action



Classification of Followers

(i) Based on surface in contact. **(Fig.3.1)**

(a) Knife edge follower

(b) Roller follower

(c) Flat faced follower

(d) Spherical follower

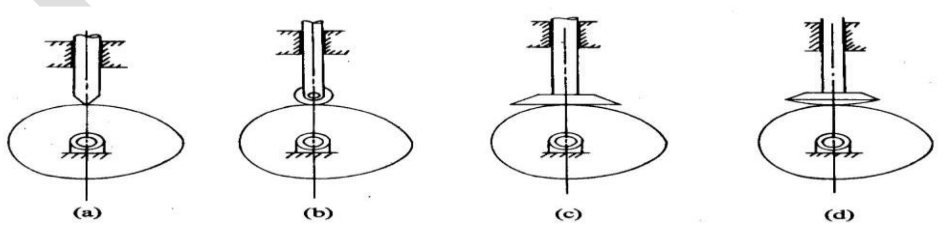


Fig. 3.1 Types of followers

(ii) Based on type of motion: **(Fig. 3.2)**

(a) Oscillating follower

(b) Translating follower

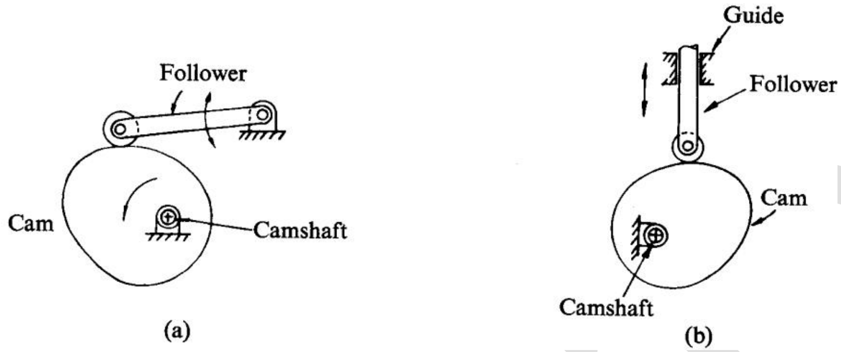


Fig.3.2

(iii) Based on line of motion:

(a) Radial follower: The lines of movement of in-line cam followers pass through the centers of the camshafts (Fig. 3.1a, b, c, and d).

(b) Off-set follower: For this type, the lines of movement are offset from the centers of the camshafts (Fig. 3.3a, b, c, and d).

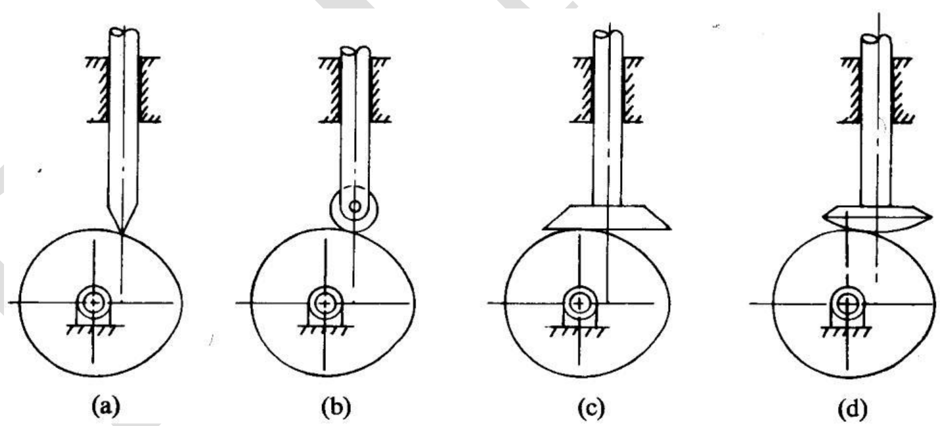


Fig.3.3 Off set followers

Classification of Cams

Cams can be classified based on their physical shape.

a) Disk or plate cam (Fig. 3.4 a and b): The disk (or plate) cam has an irregular contour to impart a specific motion to the follower. The follower moves in a plane perpendicular to the axis of rotation of the camshaft and is held in contact with the cam by springs or gravity.

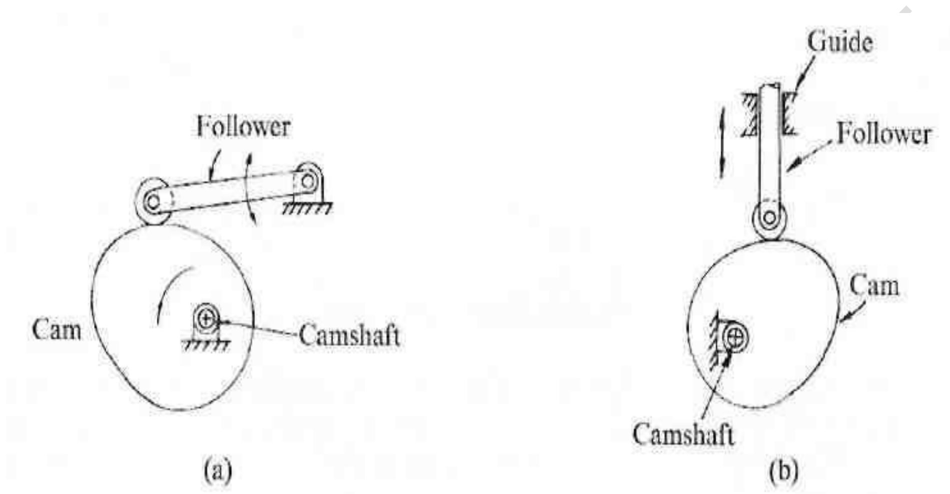


Fig. 3.4 Plate or disk cam

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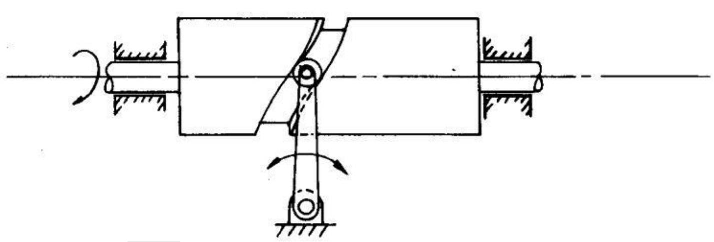
b) Cylindrical cam (Fig. 3.5): The cylindrical cam has a groove cut along its cylindrical surface. The roller follows the groove, and the follower moves in a plane



parallel to the axis of rotation of the cylinder.

Fig. 3.5 Cylindrical cam.

c) Translating cam (Fig. 3.6a and b). The translating cam is a contoured or grooved plate sliding on a guiding surface(s). The follower may oscillate (Fig. 3.6(a)) or reciprocate (Fig. 3.6(b)). The contour or the shape of the groove is determined by the specified motion of the follower.

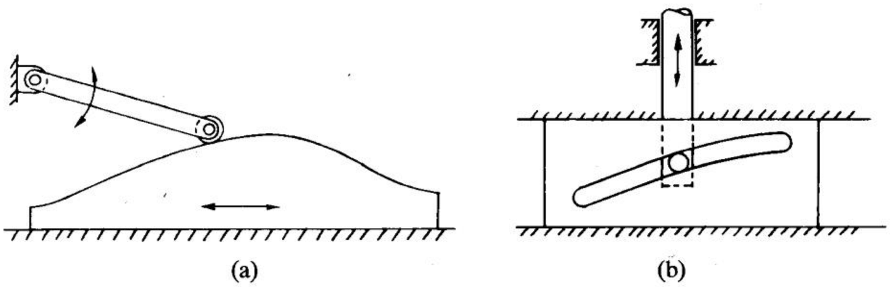


Fig. 3.6 Translating cam

Terms Used in Radial Cams

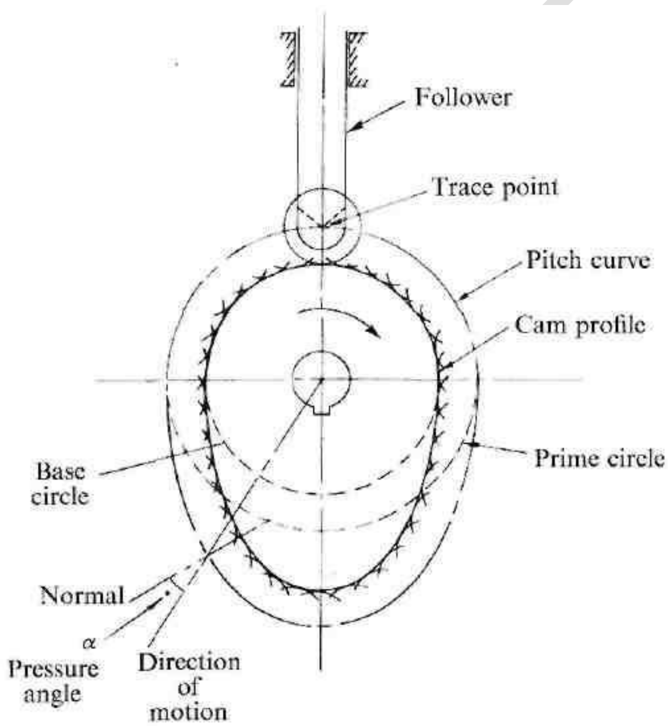


Fig.3.7

Pressure angle: It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the angle is too large, a reciprocating follower will jam in its bearings.

Base circle: It is the smallest circle that can be drawn to the cam profile.

Trace point: It is the reference point on the follower and is used to generate the pitch curve. In the case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In the roller follower, the centre of the roller represents the trace point.

Pitch point: It is a point on the pitch curve having the maximum pressure angle.

Pitch circle: It is a circle drawn from the centre of the cam through the pitch points.

Pitch curve: It is the curve generated by the trace point as the follower moves relative to the cam.For a knife edge follower, the pitch curve and the cam profile are same where as for a roller follower; they are separated by the radius of the follower.

Prime circle: It is the smallest circle that can be drawn from the centre of the cam and tangent to the point.For a knife edge and a flat face follower, the prime circle and the base circle and the base circle are identical.For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

Lift (or) stroke: It is the maximum travel of the follower from its lowest position to the topmost position.

Motion of the Follower

Cam follower systems are designed to achieve a desired oscillatory motion. Appropriate displacement patterns are to be selected for this purpose, before designing the cam surface. The cam is assumed to rotate at a constant speed and the follower raises, dwells, returns to its original position and dwells again through specified angles of rotation of the cam, during each revolution of the cam.

Some of the standard follower motions are as follows:

They are, follower motion with,

(a) Uniform velocity

(b) Modified uniform velocity

(c) Uniform acceleration and deceleration

(d) Simple harmonic motion

Displacement diagrams: In a cam follower system, the motion of the follower is very important. Its displacement can be plotted against the angular displacement θ of the cam and it is called as the displacement diagram. The displacement of the follower is plotted along the y-axis and angular displacement θ of the cam is plotted along x-axis. From the displacement diagram, velocity and acceleration of the follower can also be plotted for different angular displacements θ of the cam. The displacement, velocity and acceleration diagrams are plotted for one cycle of operation i.e., one rotation of the cam. Displacement diagrams are basic requirements for the construction of cam profiles. Construction of displacement diagrams and calculation of velocities and accelerations of followers with different types of motions are discussed in the following sections.

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity

Fig.3.8 shows the displacement, velocity and acceleration patterns of a follower having uniform velocity type of motion. Since the follower moves with constant velocity, during rise and fall, the displacement varies linearly with θ. Also, since the velocity changes from zero to a finite value, within no time, theoretically, the acceleration becomes infinite at the beginning and end of rise and fall.

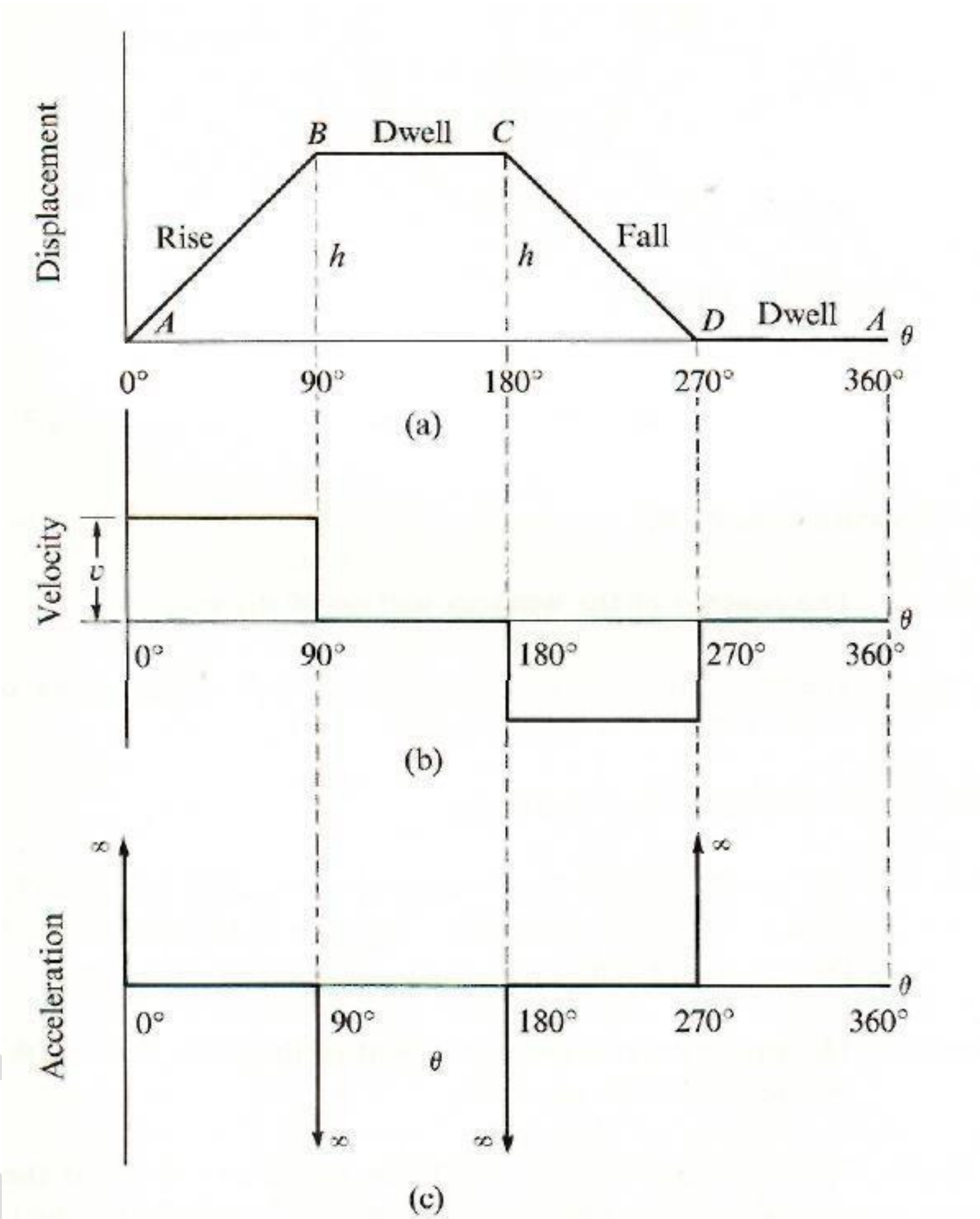


Fig.3.8

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

In fig.3.9, the motion executed by point P^l, which is the projection of point P on the vertical diameter is called simple harmonic motion. Here, P moves with uniform angular velocity ω_p, along a circle of radius r (r = s/2).

UNIT 4 BELT DRIVE

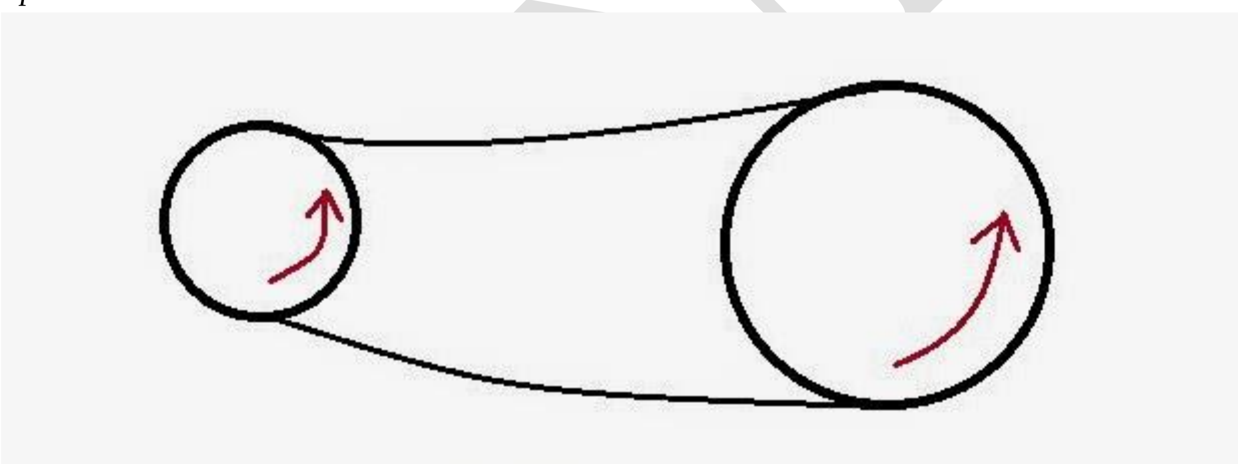
Belt Drives: Types, Advantages, Disadvantages

A belt is a looped strip of flexible material used to mechanically link two or more rotating shafts. A belt drive offers smooth transmission of power between shafts at a considerable distance. **Belt drives** are used as the source of motion to transfer to efficiently transmit power or to track relative movement.

Types of Belt Drives:

In a two pulley system, depending upon the direction the belt drives the pulley, the belt drives are divided into two types. They are open belt drive and crossed belt drive. The two types of belt drives are discussed below in brief.

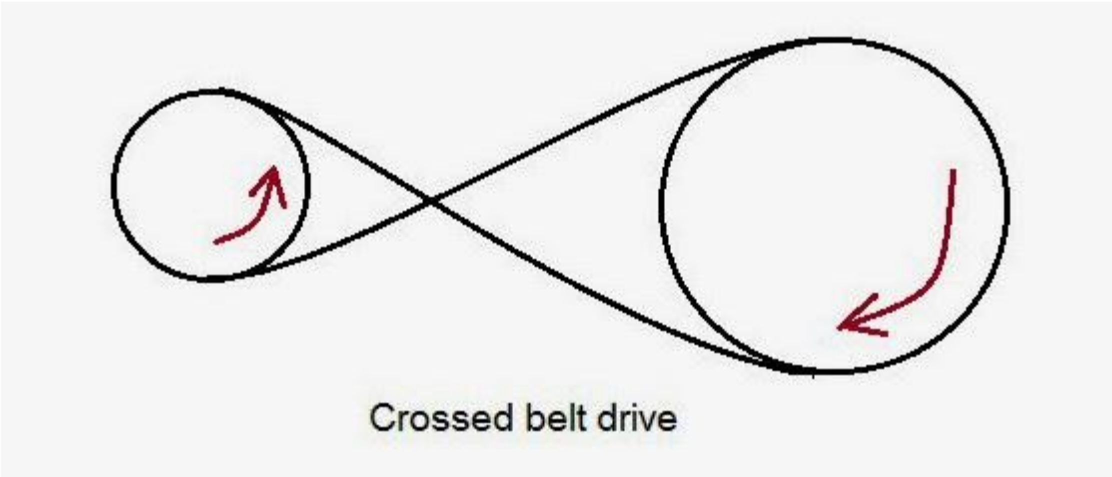
Open belt drives:



Open belt drive

An open belt drive is used to rotate the driven pulley in the same direction of driving pulley. In the motion of belt drive, power transmission results make one side of pulley more tightened compared to the other side. In horizontal drives, tightened side is always kept on the lower side of two pulleys because the sag of the upper side slightly increases the angle of folding of the belt on the two pulleys.

Crossed belt drives:



A crossed belt drive is used to rotate driven pulley in the opposite direction of driving pulley. Higher the value of wrap enables more power can be transmitted than an open belt drive. However, bending and wear of the belt are important concerns.

Advantages of belt drives:

- Belt drives are simple are economical.
- They don't need parallel shafts.
- Belts drives are provided with overload and jam protection.
- Noise and vibration are damped out. Machinery life is increased because load fluctuations are shock-absorbed.
- They are lubrication-free. They require less maintenance cost.
- Belt drives are highly efficient in use (up to 98%, usually 95%).
- They are very economical when the distance between shafts is very large.

Disadvantages of belt drives:

- In Belt drives, angular velocity ratio is not necessarily constant or equal to the ratio of pulley diameters, because of slipping and stretching.
- Heat buildup occurs. Speed is limited to usually 35 meters per second. Power transmission is limited to 370 kilowatts.
- Operating temperatures are usually restricted to –35 to 85°C.
- Some adjustment of center distance or use of an idler pulley is necessary for wearing and stretching of belt drive compensation

CLUTCH

The clutch is a mechanical device that specifically engages and disengages the power transmission from the driving shaft to the driven shaft.

In these devices, one shaft is usually connected to an engine or another power unit (driving member), while the other shaft (driven member) provides output power for the work.

The clutches used in a motor vehicle are almost very similar in construction and operation. There are some differences in the details of the linkage as well as in the pressure plate assemblies.

In addition, some clutches for heavy-duty applications has a two friction plate and an intermediate pressure plate. Some clutches are operated by hydraulic means. The dry single-plate type of friction clutch is almost used in American passenger cars.

The various types of clutches used in the automobile depend upon the type and use of friction.

Most designs of the clutches use a number of coil springs but some use a diaphragm or conical type spring. The type of friction materials also varies in the clutches of different passenger cars.

Types of Clutches

Following are the different types of clutches using in automobile industries.

1. Friction clutch
   1. Single plate clutch
   2. Multiplate clutch
      1. Wet
      2. Dry
   3. Cone clutch
      1. External
      2. Internal
2. Centrifugal Clutch
3. Semi-centrifugal clutch
4. Conical spring clutch or Diaphragm clutch
   1. Tapered finger type
   2. Crown spring type
5. Positive clutch
   1. Dog clutch

2. Spline Clutch
6. Hydraulic clutch
7. Electromagnetic clutch
8. Vacuum clutch
9. Overrunning clutch or freewheel unit

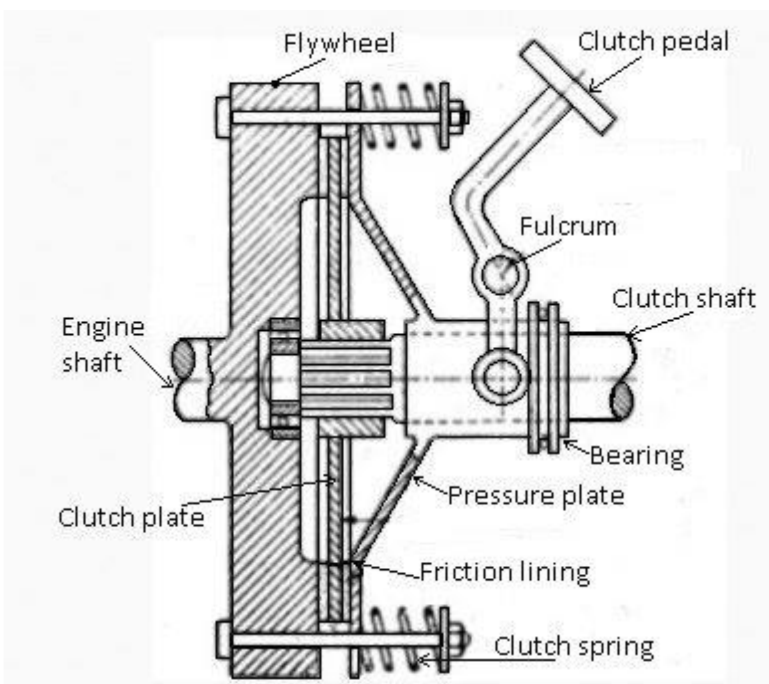
Read also: What is clutch and how it works?

Single Clutch Plate

Single plate clutches are one of the most commonly used types of clutches used in most modern light vehicles. The clutch helps to transmit torque from the engine to the transmission input shaft. As the name states it has only one clutch plate.

It consists of a clutch plate, friction plate, pressure plate, flywheel, bearings, clutch spring and not-bolts arrangement.

The single-plate clutch has only one plate which is attached on splines of the clutch plate. Single plate clutch is one of the main components of the clutch. The clutch plate is simply thin metallic disc which has both side friction surfaces.



SINGLE PLATE CLUTCH

The flywheel is attached on the engine crankshaft and rotates with it. A pressure plate is bolted to flywheel through clutch spring, which provides the axial force to keep the clutch engaged position, and is free to slide on the clutch shaft when the clutch pedal is operated.

A friction plate which is fixed between the flywheel and pressure plate. The friction lining is provided on both sides of the clutch plate.

Working:

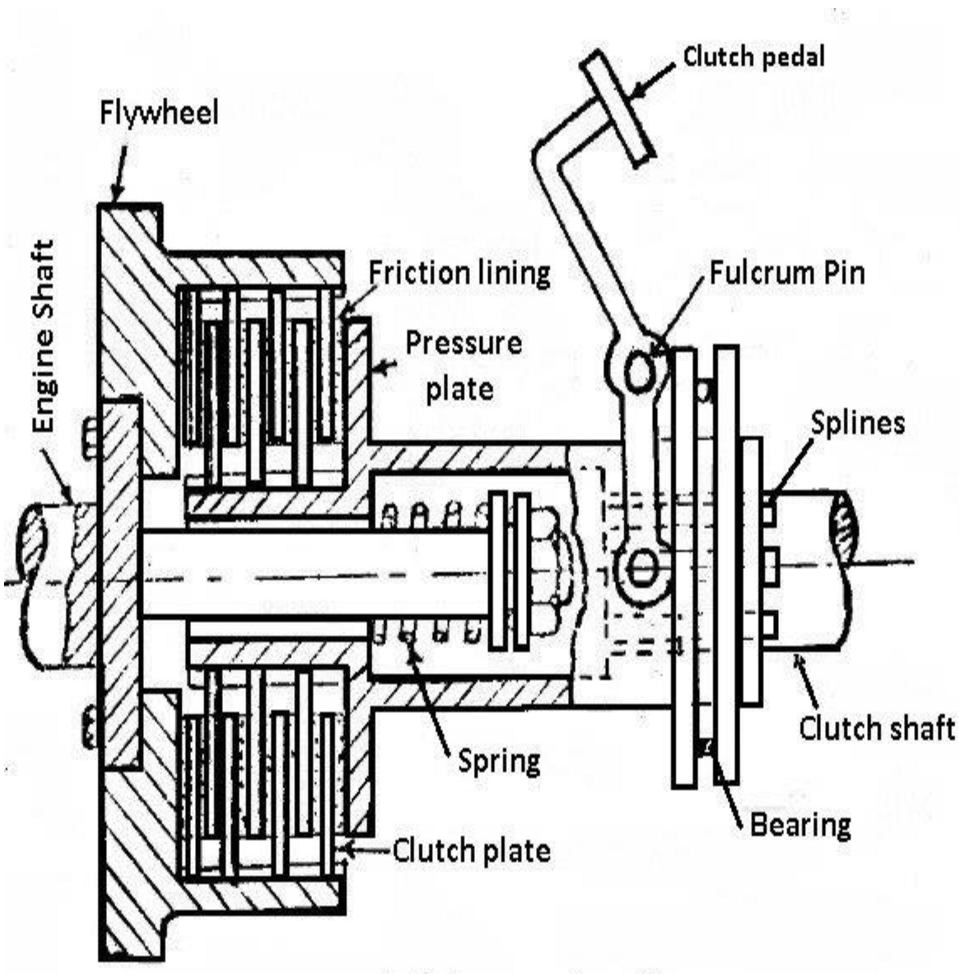
In a vehicle, we operate the clutch by pressing the clutch to peddle for disengagement of gears. Then springs get compressed and the pressure plate moves backwards. Now the clutch plate becomes free between the pressure plate and flywheel. Due to this now the clutch is getting disengaged and able to shift the gear.

This makes flywheel to rotate as long as the engine is running and the clutch shaft speed reduces slowly and then it stops rotating. As long as the clutch peddle is pressed, the clutch is said to be disengaged, otherwise, it remains engaged due to the spring forces. After releasing the clutch pedal the pressure plate comes back to its original position and clutch is again engaged.

Multiplate Clutch

The multi-plate clutch is shown in the figure. This types of clutches use multiple clutches to make frictional contact with a flywheel of the engine. This makes transmit power between the engine shaft and the transmission shaft of a vehicle. The number of clutches means more friction surface.

The increased number of friction surfaces also increases the capacity of the clutch to transmit torque. The clutch plates are fitted to the engine shaft and gearbox shaft.



Multiplate Clutch

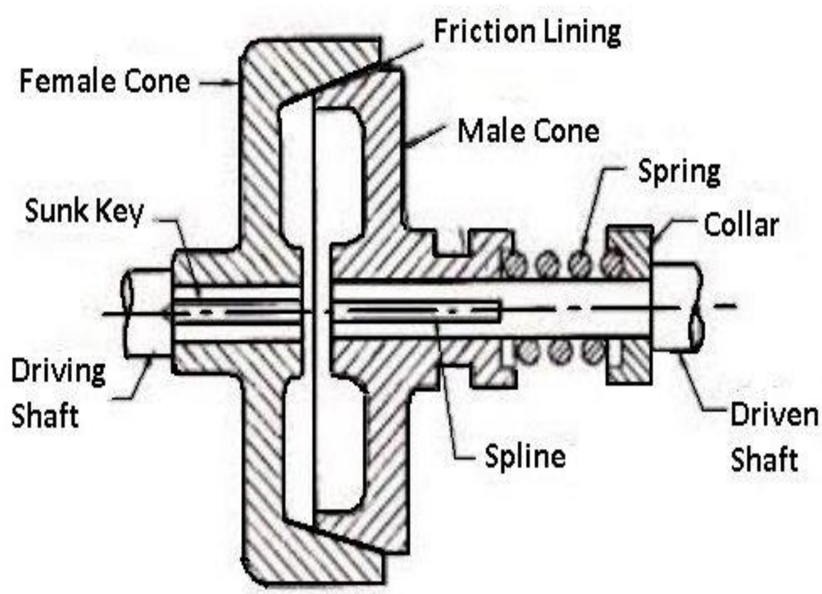
They are pressed by coil springs and assembled in a drum. Each of the alternate plates slides in grooves on the flywheel and the other slides on splines on the pressure plate. Hence, each different plate has an inner and outer spline.

The working principle of multiple clutches is the same as the working of the single-plate clutch. The clutch is operated by pressing the clutch pedal. The multiple clutches are used in heavy commercial vehicles, racing cars and motorcycles for transmitting high torque.

The multiple clutches have two characters dry and wet. If the clutch is operated in an oil bath, it is known as a wet clutch. If the clutch is operated dry without oil, it is known as a dry clutch. The wet clutches are commonly used in connection with, or as a part of the automatic transmission.

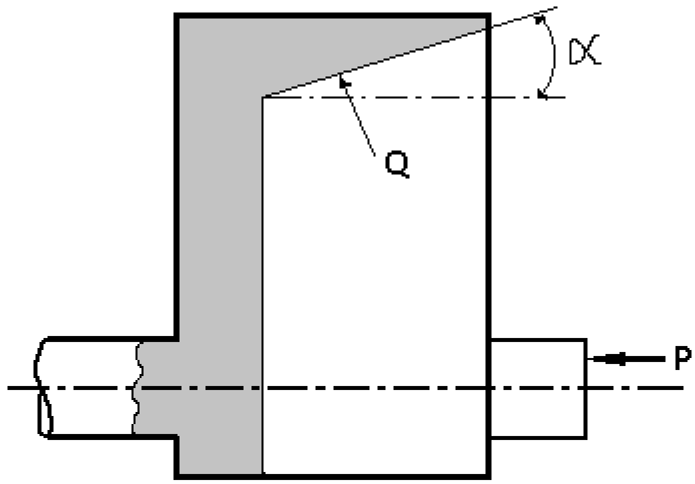
Cone Clutch

The figure shows the diagram of a cone clutch. It consists of friction surfaces in the form of cones. This clutch uses two conical surfaces to transmit torque by friction. The engine shaft consists of a female cone and a male cone. The male cone is mounted on the splined clutch shaft to slide on it. It has a friction surface on the conical portion.



Cone Clutch

Due to the force of spring when the clutch is engaged the friction surfaces of the male cone are in contact with the female cone. When the clutch pedal is pressed, the male cone slides towards the spring force and the clutch is disengaged.



Force on a Cone Clutch

If

$\propto = Semi\ cone\ angle$

$P = Axial\ force$

$Q = Normal\ force$

Then $Q = \dfrac{p}{\cos \propto}$

Which is greater than P.

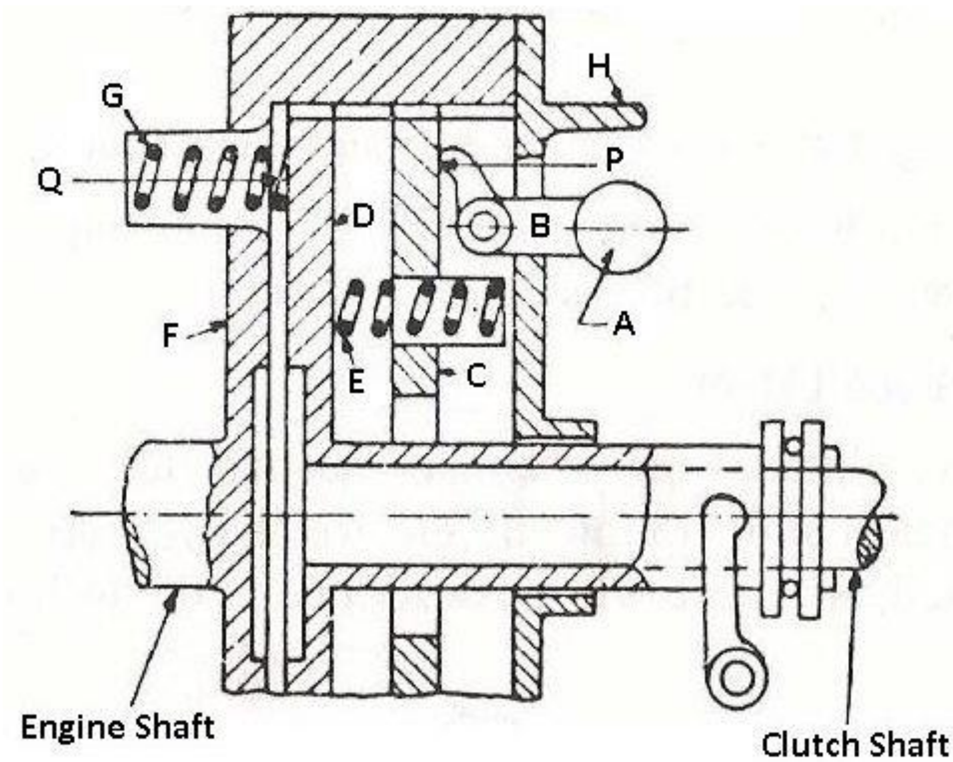
The main advantage of using cone clutch is that the normal force acting on the friction surface is greater than the axial force, as compared to the single-plate clutch. That’s why the normal force acting on the friction surface is equal to the axial force.

The cone clutches are basically becoming old because of **some disadvantages.**

1. Let’s consider the angle of the cone is made smaller than 20°, the male cone tends to bind in the female cone and it becomes difficult to disengage the clutch.
2. A small amount of wear on the cone surfaces has a considerable amount of axial movement of male cones, for which it will be difficult to allow it.

Centrifugal Clutch

The below figure shows a centrifugal clutch. To keep the clutches in the engaged position centrifugal clutch uses centrifugal force, instead of spring force. In these types of clutches, the clutch is operated automatically depending upon the engine speed. That's why no clutch pedal is required to operate the clutch.



CENTRIFUGAL CLUTCH

This made so easy for the driver to stop the vehicle in any gear without stalling the engine. Similarly, you can start the vehicle in any gear by pressing the accelerator pedal.

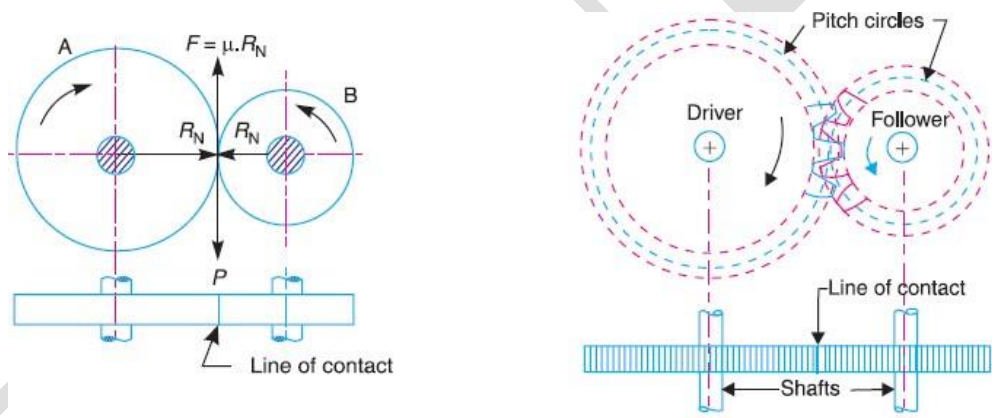
Working of Centrifugal clutch

- It consists of weights A pivoted at B.
- When the engine speed increases the weights fly off due to the centrifugal force, operating the bell crank levels, which press the plate C.
- The movement of plate C presses the spring E, Which ultimately presses the clutch plate D on the flywheel against the spring G.
- This makes the clutch engaged.
- The spring G keeps the clutch disengaged at low speeds at about 500rpm.
- The stop H limits the movement of the weights due to the centrifugal

Unit:V GEARS

Gears are machine elements that transmit motion by means of successively engaging teeth. The gear teeth act like small levers. Gears are highly efficient (nearly 95%) due to primarily rolling contact between the teeth, thus the motion transmitted is considered as positive. Gears essentially allow positive engagement between teeth so high forces can be transmitted while still undergoing essentially rolling contact. Gears do not depend on friction and do best when friction is minimized.

Let the wheel A be keyed to the rotating shaft and the wheel B to the shaft, to be rotated. A little consideration will show, that when the wheel A is rotated by a rotating shaft, it will rotate the wheel B in the opposite direction as shown in Fig. 4.1 (a). The wheel B will be rotated (by the wheel A) so long as the tangential force exerted by the wheel A does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (P) exceeds the frictional resistance (F), slipping will take place between the two wheels. Thus the friction drive is not a positive drive.



(*a*) Friction wheels. (*b*) Toothed wheels.

Fig. 4.1.

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. 5.1 (b), are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B. A friction wheel with the teeth cut on it is known as **toothed wheel or gear.** The usual connection to show the toothed wheels is by their pitch circles.

Advantages and Disadvantages of Gear Drive

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages:

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages:

1. The manufacture of gears requires special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

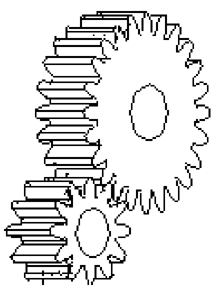
Classification of Toothed Wheels

Gears may be classified according to the relative position of the axes of revolution. The axes may be

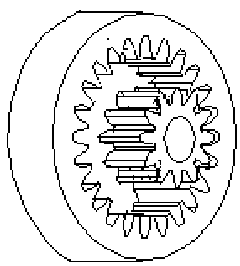
1. Gears for connecting parallel shafts,
2. Gears for connecting intersecting shafts,
3. Gears for neither parallel nor intersecting shafts.

Gears for connecting parallel shafts

1. **Spur gears:** Spur gears are the most common type of gears. They have straight teeth, and are mounted on parallel shafts. Sometimes, many spur gears are used at once to create very large gear reductions. Each time a gear tooth engages a tooth on the other gear, the teeth collide, and this impact makes a noise. It also increases the stress on the gear teeth. To reduce the noise and stress in the gears, most of the gears in your car are



External contact



Internal contact

Spur gears

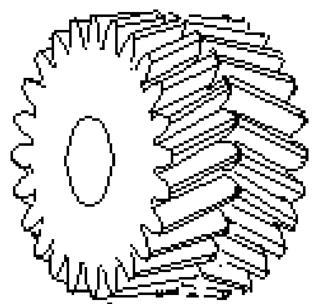
Spur gears are the most commonly used gear type. They are characterized by teeth, which are perpendicular to the face of the gear. Spur gears are most commonly available, and are generally the least expensive.

- **Limitations:** Spur gears generally cannot be used when a direction change between the two shafts is required.
- **Advantages:** Spur gears are easy to find, inexpensive, and efficient.

2. **Parallel helical gears:** The teeth on helical gears are cut at an angle to the face of the gear. When two teeth on a helical gear system engage, the contact starts at one end of the tooth and gradually spreads as the gears rotate, until the two teeth are in full engagement.

epared by Kiran K

Helical gears



Herringbone gears
(or double-helical gears)

This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears. For this reason, helical gears are used in almost all car transmission.

Because of the angle of the teeth on helical gears, they create a thrust load on the gear when they mesh. Devices that use helical gears have bearings that can support this thrust load.

One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle by 90 degrees.

Helical gears to have the following differences from spur gears of the same size:

- Tooth strength is greater because the teeth are longer,
- Greater surface contact on the teeth allows a helical gear to carry more load than a spur gear
- The longer surface of contact reduces the efficiency of a helical gear relative to a spur gear

Rack* and *pinion: (The rack is like a gear whose axis is at infinity mathematically but practically a gear of larger length.)

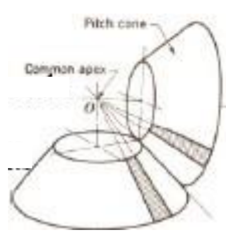
Racks are straight gears that are used to convert rotational motion to translational motion by means of a gear mesh. (They are in theory a gear with an infinite pitch diameter).
In theory, the torque and angular velocity of the pinion gear are related to the Force and the velocity of the rack by the radius of the pinion gear, as is shown.

Perhaps the most well-known application of a rack is the rack and pinion steering system used on many cars in the past.

Gears for connecting intersecting shafts: Bevel gears are useful when the direction of a shaft's rotation needs to be changed. They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well.

The teeth on bevel gears can be straight, spiral or hypoid. Straight bevel gear teeth actually have the same problem as straight spur gear teeth, as each tooth engages; it impacts the corresponding tooth all at once.

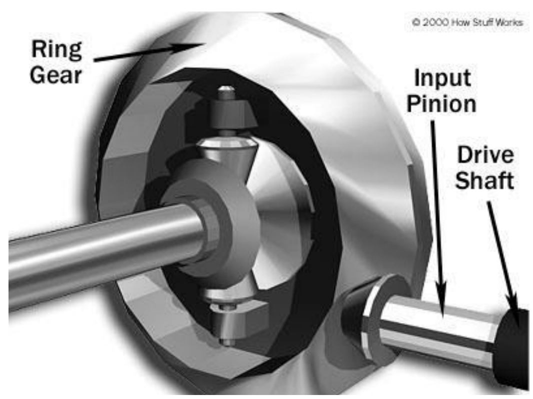
Just like with spur gears, the solution to this problem is to curve the gear teeth. These spiral teeth engage just like helical teeth: the contact starts at one end of the gear and progressively spreads across the whole tooth.





Straight bevel gears *Spiral bevel gears*

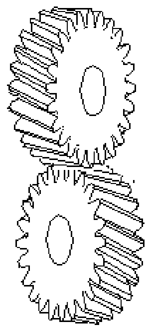
On straight and spiral bevel gears, the shafts must be perpendicular to each other, but they must also be in the same plane. The hypoid gear, can engage with the axes in different planes.



Hypoid gears

This feature is used in many car differentials. The ring gear of the differential and the input pinion gear are both hypoid. This allows the input pinion to be mounted lower than the axis of the ring gear. Figure shows the input pinion engaging the ring gear of the differential. Since the driveshaft of the car is connected to the input pinion, this also lowers the driveshaft. This means that the driveshaft doesn't pass into the passenger compartment of the car as much, making more room for people and cargo.

Neither parallel nor intersecting shafts: Helical gears may be used to mesh two shafts that are not parallel, although they are still primarily use in parallel shaft applications. A special application in which helical gears are used is a crossed gear mesh, in which the two shafts are perpendicular to each other.



Crossed-helical gears

Worm and worm gear*:* Worm gears are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater.

Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm. This is because the angle on the worm is so shallow that when the gear tries to spin it, the friction between the gear and the worm holds the worm in place.

This feature is useful for machines such as conveyor systems, in which the locking feature can act as a brake for the conveyor when the motor is not turning. One other very interesting usage of worm gears is in the Torsen differential, which is used on some high-performance cars and trucks.

Terms Used in Gears

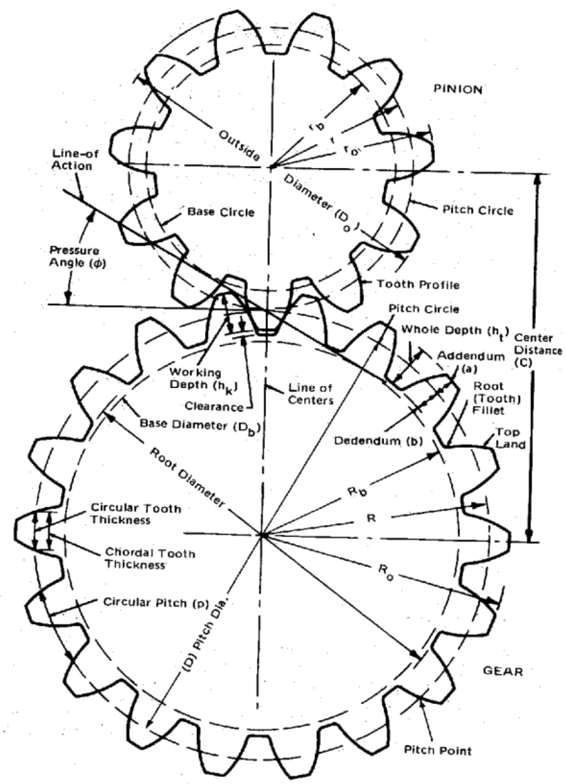
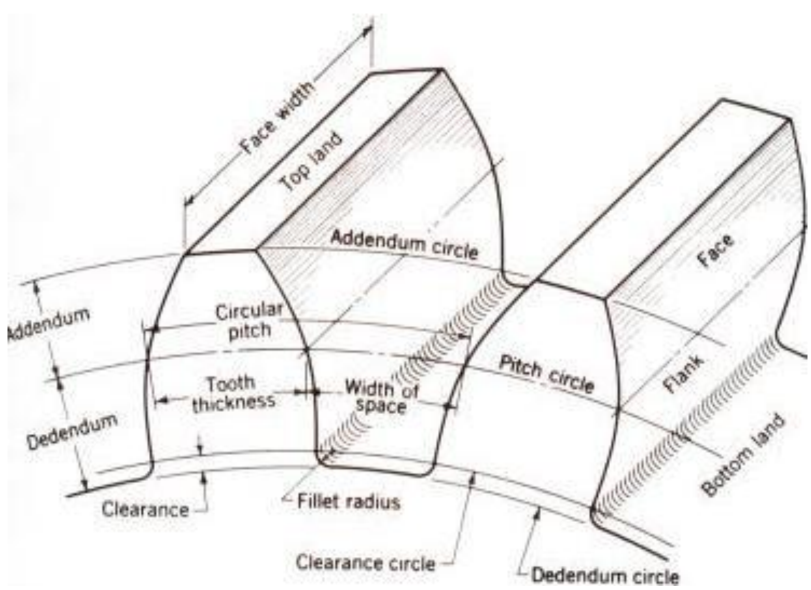


Fig. 4.2. Spur Gear and Pinion pair

Terms used in Gear



Addendum: The radial distance between the Pitch Circle and the top of the teeth.

Dedendum: The radial distance between the bottom of the tooth to pitch circle.

Base Circle: The circle from which is generated the involute curve upon which the tooth profile is based.

Center Distance: The distance between centers of two gears.

Circular Pitch: Millimeter of Pitch Circle circumference per tooth.

Circular Thickness: The thickness of the tooth measured along an arc following the Pitch Circle

Clearance: The distance between the top of a tooth and the bottom of the space into which it fits on the meshing gear.

Contact Ratio: The ratio of the length of the Arc of Action to the Circular Pitch.

Diametral Pitch: Teeth per mm of diameter.

Face: The working surface of a gear tooth, located between the pitch diameter and the top of the tooth.

Face Width: The width of the tooth measured parallel to the gear axis.

Flank: The working surface of a gear tooth, located between the pitch diameter and the bottom of the teeth

Gear: The larger of two meshed gears. If both gears are the same size, they are both called "gears".

Land: The top surface of the tooth.

Line of Action: That line along which the point of contact between gear teeth travels, between the first point of contact and the last.

Module: Millimeter of Pitch Diameter to Teeth.

Pinion: The smaller of two meshed gears.

Pitch Circle: The circle, the radius of which is equal to the distance from the center of the gear to the pitch point.

Diametral pitch: Teeth per millimeter of pitch diameter.

Pitch Point: The point of tangency of the pitch circles of two meshing gears, where the Line of Centers crosses the pitch circles.

Pressure Angle: Angle between the Line of Action and a line perpendicular to the Line of Centers.

Root Circle: The circle that passes through the bottom of the tooth spaces.

Working Depth: The depth to which a tooth extends into the space between teeth on the mating gear.

Gear-Tooth Action

Fundamental Law of Gear-Tooth Action

Figure shows two mating gear teeth, in which

- Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point *K*.
- N_1N_2 is the common normal of the two profiles.
- N_1 is the foot of the perpendicular from O_1 to N_1N_2
- N_2 is the foot of the perpendicular from O_2 to N_1N_2.

Although the two profiles have different velocities $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$ at point *K*, their velocities along N_1N_2 are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, we have

$$O_1N_1 \omega_1 = O_2N_2 \omega_2$$

or

$$\frac{\omega_1}{\omega_2} = \frac{O_2N_2}{O_1N_1}$$

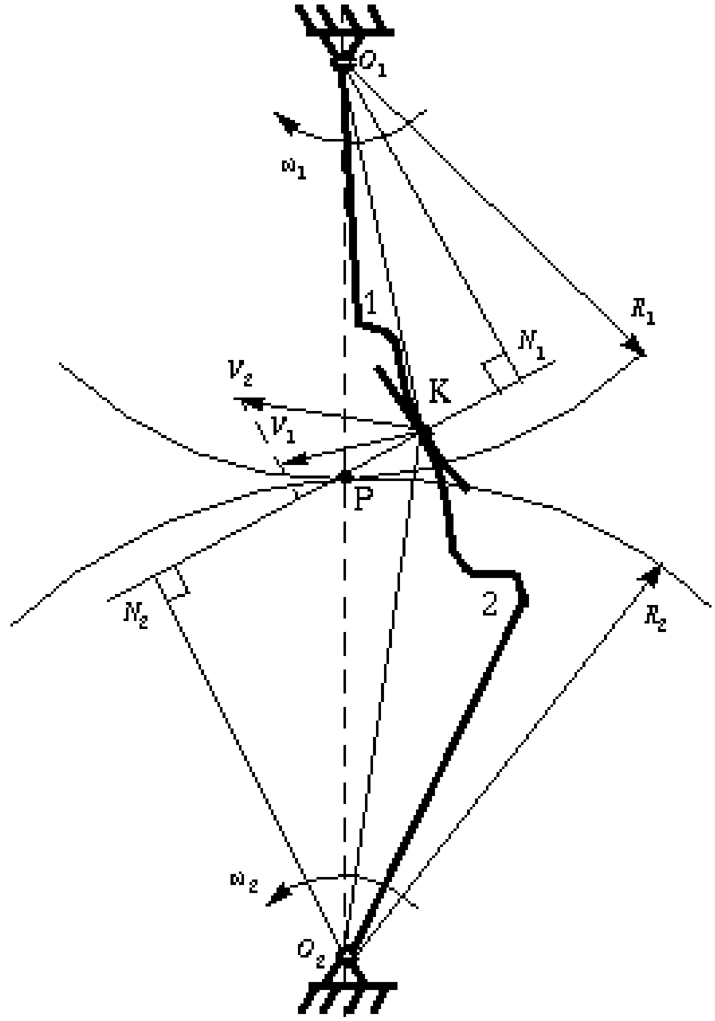


Figure 5-2 Two gearing tooth profiles

We notice that the intersection of the tangency N_1N_2 and the line of center O_1O_2 is point *P*, and from the similar triangles,

$$\triangle O_1N_1P \sim \triangle O_2N_2P$$

Thus, the relationship between the angular velocities of the driving gear to the driven gear, or **velocity ratio**, of a pair of mating teeth is

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

Point P is very important to the velocity ratio, and it is called the **pitch point**. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The above expression is the **fundamental law of gear-tooth action**.

Constant Velocity Ratio

For a constant velocity ratio, the position of P should remain unchanged. In this case, the motion transmission between two gears is equivalent to the motion transmission between two imagined slip-less cylinders with radius R_1 and R_2 or diameter D_1 and D_2. We can get two circles whose centers are at O_1 and O_2, and through pitch point P. These two circles are termed **pitch circles**. The velocity ratio is equal to the inverse ratio of the diameters of pitch circles. This is the fundamental law of gear-tooth action.

The **fundamental law of gear-tooth action** may now also be stated as follow (for gears with fixed center distance)

A common normal (the line of action) to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point.
Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves, and the relative rotation speed of the gears will be constant(constant velocity ratio).

Conjugate Profiles

To obtain the expected *velocity ratio* of two tooth profiles, the normal line of their profiles must pass through the corresponding pitch point, which is decided by the *velocity ratio*. The two profiles which satisfy this requirement are called **conjugate profiles**. Sometimes, we simply termed the tooth profiles which satisfy the *fundamental law of gear-tooth action* the *conjugate profiles*.

Although many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the *cycloidal* and *involute* profiles. The involute has important advantages; it is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required when using the involute profile. The most commonly used *conjugate* tooth curve is the *involute curve.*

conjugate action : It is essential for correctly meshing gears, the size of the teeth (the module) must be the same for both the gears.

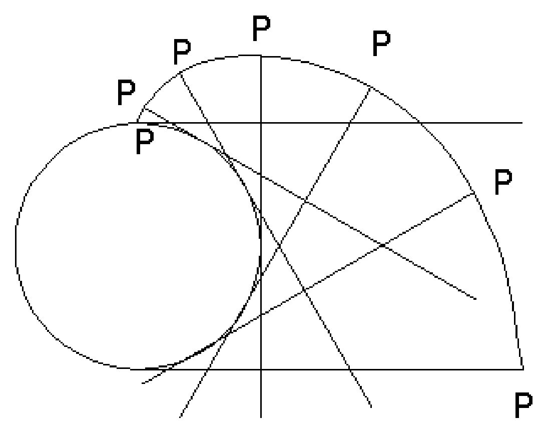
Another requirement - the shape of teeth necessary for the speed ratio to remain constant during an increment of rotation; this behavior of the contacting surfaces (ie. the teeth flanks) is known as ***conjugate action.***

Forms of Teeth
Involute Profile

The following examples are involute spur gears. We use the word *involute* because the contour of gear teeth curves inward. Gears have many terminologies, parameters and principles. One of the important concepts is the *velocity ratio,* which is the ratio of the rotary velocity of the driver gear to that of the driven gears.

Generation of the Involute Curve

The curve most commonly used for gear-tooth profiles is the involute of a circle. This **involute curve** is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string, which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the **base circle**.



Involute curve

The involute profile of gears has important advantages;

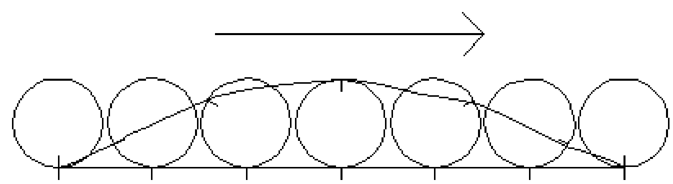
1. It is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required. The most commonly used *conjugate* tooth curve is the *involute curve.*

2. In involute gears, the pressure angle, remains constant between the point of tooth engagement and disengagement. It is necessary for smooth running and less wear of gears.

3. The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (i.e. epi-cycloid and hypo-cycloid) are required for the face and flank respectively. Thus the involute teeth are easy to manufacture than cycloidal teeth.

In involute system, the basic rack has straight teeth and the same can be cut with simple tools.

Cycloidal profile:



A ***cycloid*** is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as ***epi-cycloid***. On the

other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called ***hypo-cycloid***.

Advantages of Cycloidal gear teeth:

1. Since the cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears, for the same pitch. Due to this reason, the cycloidal teeth are preferred specially for cast teeth.

2. In cycloidal gears, the contact takes place between a convex flank and a concave surface, where as in involute gears the convex surfaces are in contact. This condition results in less wear in cycloidal gears as compared to involute gears. However the difference in wear is negligible

3. In cycloidal gears, the interference does not occur at all. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

S.No.	Involute tooth gears	Cycloid tooth gears
1.	The profile of involute gears is the single curvature.	The profile of cycloidal gears is double curvature i.e. epicycloid and hypocycloid.
2.	The pressure angle from start of engagement of teeth to the end of engagement remains constant, which results into smooth running.	The pressure angle varies from start of engagement to end of engagement, which results into less smooth running.
3.	Manufacturing of involute gears is easy due to single curvature of tooth profile.	Manufacturing of cycloidal gears is difficult due to double curvature of tooth profile.
4.	The involute gears have interference problem.	The cycloidal gears do not have interference problem.
5.	More wear of tooth surface.	Less wear as convex face engages with concave flank.
6.	The strength of involute teeth is less due to radial flanks	The strength of cyloidal teeth is comparatively more due to wider flanks.

Systems of Gear Teeth

The following four systems of gear teeth are commonly used in practice:

1. 14 ½^{O} Composite system
2. 14 ½^{O} Full depth involute system
3. 20^{O} Full depth involute system
4. 20^{O} Stub involute system

The 14½^{O} *composite system* is used for general purpose gears.

It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion.

The teeth are produced by formed milling cutters or hobs.

The tooth profile of the 14½^{O} *full depth involute system* was developed using gear hobs for spur and helical gears.

The tooth profile of the 20^{o} *full depth involute system* may be cut by hobs.

The increase of the pressure angle from 14½^{o} to 20^{o} results in a stronger tooth, because the tooth acting as a beam is wider at the base.

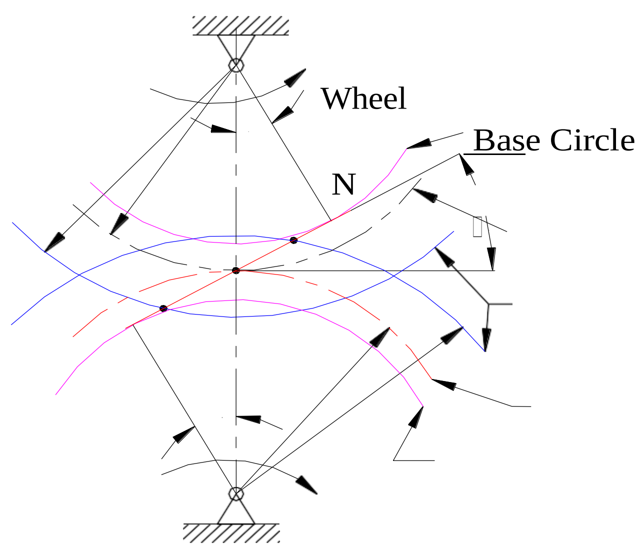
The *20^{o} stub involute system* has a strong tooth to take heavy loads.

Length of Path of Contact

O2

RA

R



Consider a pinion driving wheel as shown in figure. When the pinion rotates in clockwise, the contact between a pair of involute teeth begins at *K* (on the near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at *L* (outer end of the tooth face on the pinion or on the flank near the base circle of wheel).

MN is the common normal at the point of contacts and the common tangent to the base circles. The point *K* is the intersection of the addendum circle of wheel and the common tangent. The point *L* is the intersection of the addendum circle of pinion and common tangent.

The length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion. Thus the length of part of contact is *KL* which is the sum of the parts of path of contacts *KP* and *PL*. Contact length *KP* is called as **path of approach** and contact length *PL* is called as **path of recess**.

$r_a = O_1L$ = Radius of addendum circle of pinion,and

$R_A = O_2K$ = Radius of addendum circle of wheel

$r = O_1P$ = Radius of pitch circle of pinion,

and $R = O_2P$ = Radius of pitch circle of wheel.

Radius of the base circle of pinion = $O_1M = O_1P\cos\phi = r\cos\phi$

and radius of the base circle of wheel = $O2N = O2P\cos\phi = R\cos\phi$

From right angle triangle O_2KN

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2}$$

$$= \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$PN = O_2P\sin\phi = R\sin\phi$$

Path of approach: *KP*

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly from right angle triangle O_1ML

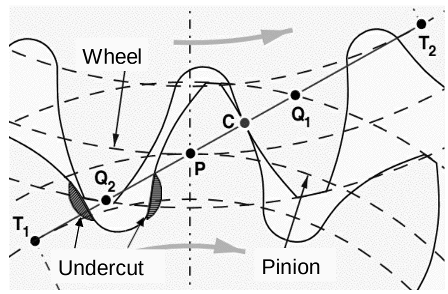
$$ML = \sqrt{(O_1L)^2 - (O_1M)^2}$$

$$= \sqrt{(r_a)^2 - r^2\cos^2\phi}$$

Interference in Involute Gears

The tooth tip of the pinion will then undercut the tooth on the wheel at the root and damages part of the involute profile. This effect is known as *interference*, and occurs when the teeth are being cut and weakens the tooth at its root.

In general, the phenomenon, when the tip of tooth undercuts the root on its mating gear is known as interference.



Similarly, if the radius of the addendum circles of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called interference points. The interference may only be prevented, if the point of contact between the two teeth is always on the involute profiles and if the addendum circles of the two mating gears cut the common tangent to the base circles at the points of tangency.

1. Height of the teeth may be reduced.
2. Under cut of the radial flank of the pinion.
3. Centre distance may be increased. It leads to increase in pressure angle.

Minimum number of teeth on the pinion avoid Interference 't'

$$t = \frac{2a_p}{\left[\left(1 + G(G+2)\sin^2\phi\right)^{\frac{1}{2}} - 1\right]}$$

Minimum number of teeth on the wheel avoid Interference 'T'

$$T = \frac{2a_w}{\left[\left(1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi\right)^{\frac{1}{2}} - 1\right]}$$

Backlash:

The gap between the non-drive face of the pinion tooth and the adjacent wheel tooth is known as ***backlash***. Backlash is the error in motion that occurs when gears change direction. The term "backlash" can also be used to refer to the size of the gap, not just the phenomenon it causes; thus, one could speak of a pair of gears as having, for example, "0.1 mm of backlash."

Practise problems:

1) Two gears in mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 teeth while the pinion has 23 teeth. If the addenda on pinion and gear wheel are equal to one module (1*m*), find

a. The number of pairs of teeth in contact and

b. The angle of action of the pinion and the gear wheel.

Solution:

Data: t =23; T=57; addendum = 1m= $8mm$ and f=20°

$$\text{Pitch circle radius of the pinion} = r = \frac{mt}{2} = \frac{8 \times 23}{2} = 92mm$$

$$\text{Pitch circle radius of the gear} = R = \frac{mT}{2} = \frac{8 \times 57}{2} = 228mm$$

$$\text{Addendum circle radius of the pinion} = r_a = r + \text{addendum}$$

$$r_a = 92 + 8 = 100mm$$

$$\text{Addendum circle radius of the gear} = R_A = R + \text{addendum}$$

$$R_A = 228 + 8 = 236mm$$

$$\text{Length of path of contact} = KL = KP + PL$$

$$= \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_a)^2 - r^2\cos^2\phi} - (R + r)\sin\varphi$$

$$= \sqrt{(236)^2 - (228)^2\cos^2 20} + \sqrt{(100)^2 - (92)^2\cos^2 20} - (228 + 92)\sin 20$$

$$= 39.76mm$$

$$\text{Length of arc of contact} = \frac{\text{Length of path of contact}}{\cos\varphi}$$

$$= \frac{39.76}{\cos 20} = 42.31mm$$

$$\text{Number of pairs of teeth in contact} = \frac{\text{Length of arc of contact}}{\text{circular pitch}}$$