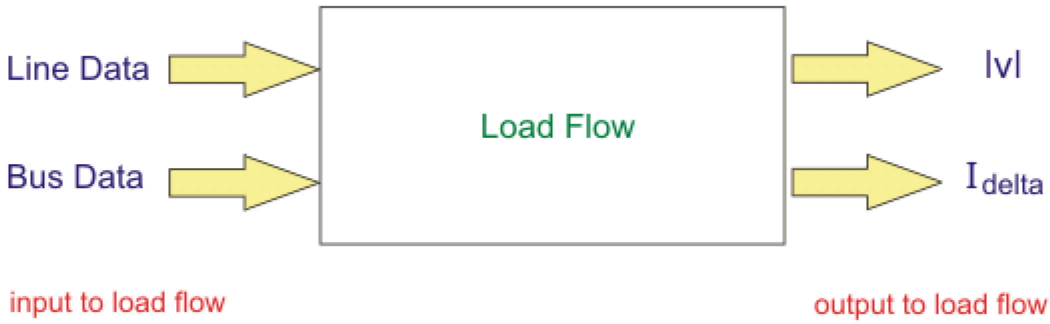
Load Flow or Power Flow Analysis
It is the computational procedure (numerical algorithms) required to determine the steady state operating characteristics of a power system network from the given line data and bus data.



Things you must know about load flow:

1. **Load flow** study is the steady state analysis of power system network.
2. Load flow study determines the operating state of the system for a given loading.
3. Load flow solves a set of simultaneous non linear algebraic power equations for the two unknown variables (|V| and ∠δ) at each node in a system.
4. To solve non linear algebraic equations it is important to have fast, efficient and accurate numerical algorithms.
5. The output of the load flow analysis is the voltage and phase angle, real and reactive power (both sides in each line), line losses and slack bus power.

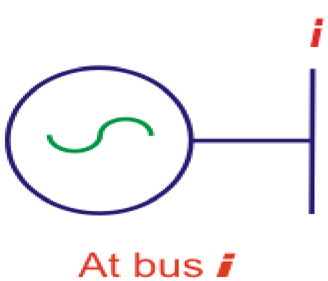
Load Flow Steps

The study of load flow involves the following three steps:

1. Modeling of power system components and network.
2. Development of load flow equations.
3. Solving the **load flow** equations using numerical techniques.

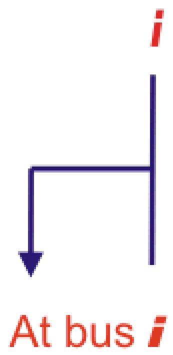
Modeling of Power System Components

Generator



$$S_{Gi} = P_{Gi} + jQ_{Gi} \quad (total\ generation\ at\ bus\ i)$$

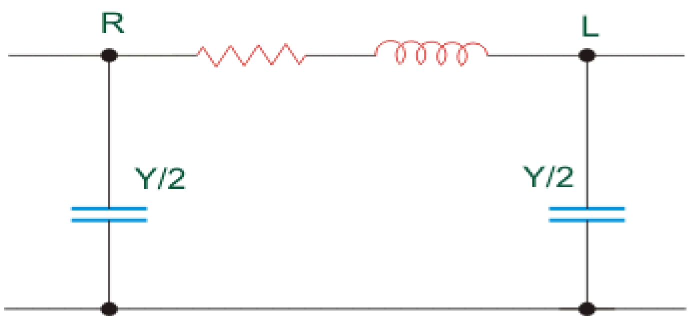
Load



$$S_{Di} = P_{Di} + jQ_{Di} \quad (total\ demand\ at\ bus\ i)$$

Transmission Line

A Transmission line is represented as a nominal π model.



Where, R + jX is the line impedance and Y/2 is called the half line charging admittance.

Off Nominal Tap Changing Transformer

For a nominal transformer the relation $\frac{N_1}{N_2} = \frac{E_1}{E_2}$ *holds true.*

But for an off nominal transformer

$$\frac{N_1}{N_2} \neq \frac{E_1}{E_2}$$

Thus for an off nominal transformer we define the transformation ratio (a) as follows

$$Transformation\ ratio\ (a) = Tap\ ratio/Nominal\ ratio$$

Now we would like to represent an off nominal transformer in a line by an equivalent model.

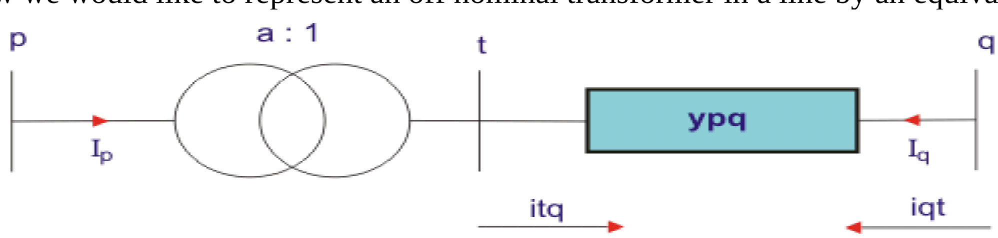


Fig 2: Line Containing an Off Nominal Transformer

We want to convert the above into an equivalent π model between bus p and q.

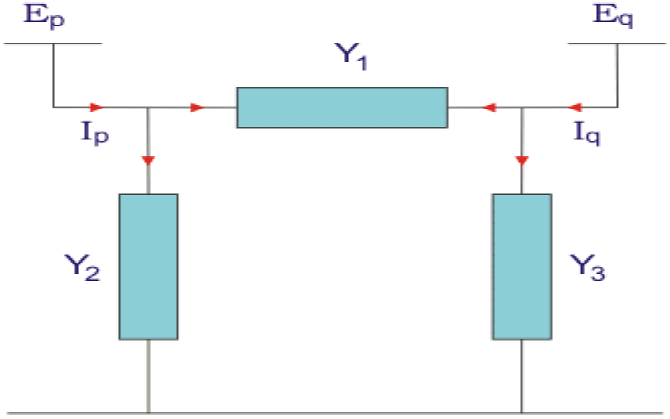


Fig 3: Equivalent π Model of Line

Our aim is to find these values of admittances Y_1, Y_2 and Y_3 so that fig2 can be represented by fig 3

From Fig 2 we have,

$$\frac{I_p}{I_{tq}} = \frac{1}{a} \Rightarrow I_p = \frac{I_{tq}}{a}$$

$$\frac{E_p}{E_t} = \frac{a}{1} \Rightarrow E_t = \frac{E_p}{a}$$

$$I_p = \frac{I_{tq}}{a} = (E_t - E_q) \times \frac{y_{pq}}{a}$$

$$I_p = (E_p - a \times E_q) \times \frac{y_{pq}}{a^2} \rightarrow (I)$$

$$I_q = (E_q - E_t) \times y_{pq}$$

$$I_q = (a \times E_q - E_p) \times \frac{y_{pq}}{a} \rightarrow (II)$$

Now consider Fig 3, from fig3 we have,

$$I_p = E_p \times Y_2 + (E_p - E_q) \times Y_1 \rightarrow (III)$$

$$I_q = E_q \times Y_3 + (E_q - E_p) \times Y_1 \rightarrow (IV)$$

From eqn I and III on comparing the coefficients of E_p and E_q we get,

$$Y_1 = \frac{y_{pq}}{a}$$

$$Y_2 = \frac{1}{a} \times (\frac{1}{a} - 1) \times y_{pq}$$

Similarly from equation II and IV we have

$$Y_3 = (1 - \frac{1}{a}) \times y_{pq}$$

Some useful observations

$$\bullet If\ a = 1, Y_1 = y_{pq}, Y_2 = 0, Y_3 = 0.$$

$$\bullet\ If\ a > 1, Y_1 = \frac{y_{pq}}{a}, Y_2 = -ve, Y_3 = +ve.$$

$$\bullet If\ a < 1, Y_1 = \frac{y_{pq}}{a}, Y_2 = +ve, Y_3 = -ve.$$

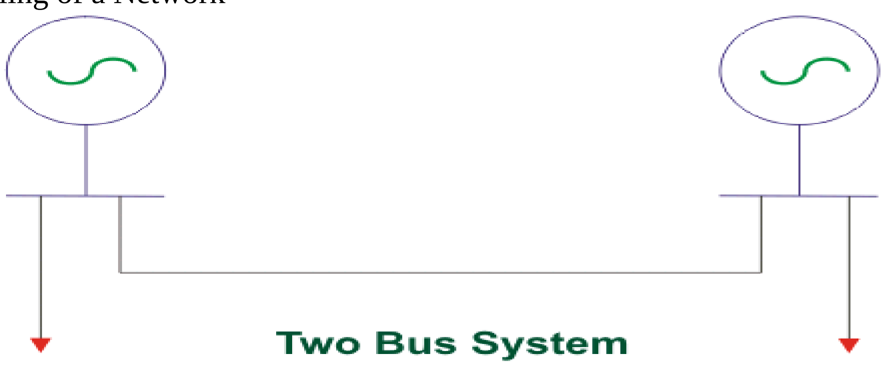
From above analysis we see that Y_2, Y_3 values can either be positive or negative depending on the value of transformation ratio.

Good question!

Y = – ve implies absorption of reactive power i.e it is behaving as an inductor.

Y = + ve implies generation of reactive power i.e it is behaving as a capacitor.

Modeling of a Network



Consider the two bus system as shown in figure above.

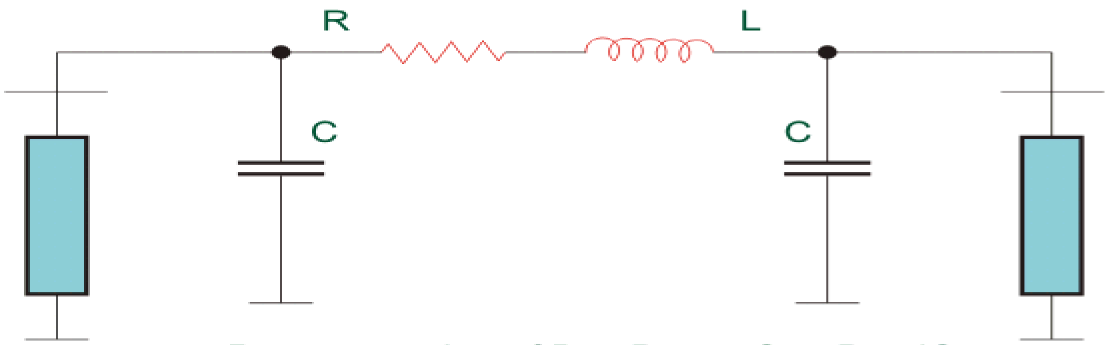
We have already seen that

Power generated at bus i is

$$S_{Gi} = P_{Gi} + jQ_{Gi}$$

Power demand at bus i is

$$S_{Di} = P_{Di} + jQ_{Di}$$



Representation of Bus Power $S_i = P_i + jQ_i$

Therefore we define the net power injected at bus i as follows

$$S_i = S_{Gi} - S_{Di} = (P_{Gi} - P_{Di}) + j(Q_{Gi} - Q_{Di}) = P_i + jQ_i$$
$$= net\ power\ injected\ at\ bus\ i$$

Formation of Bus Admittance Matrix (Y_{bus})

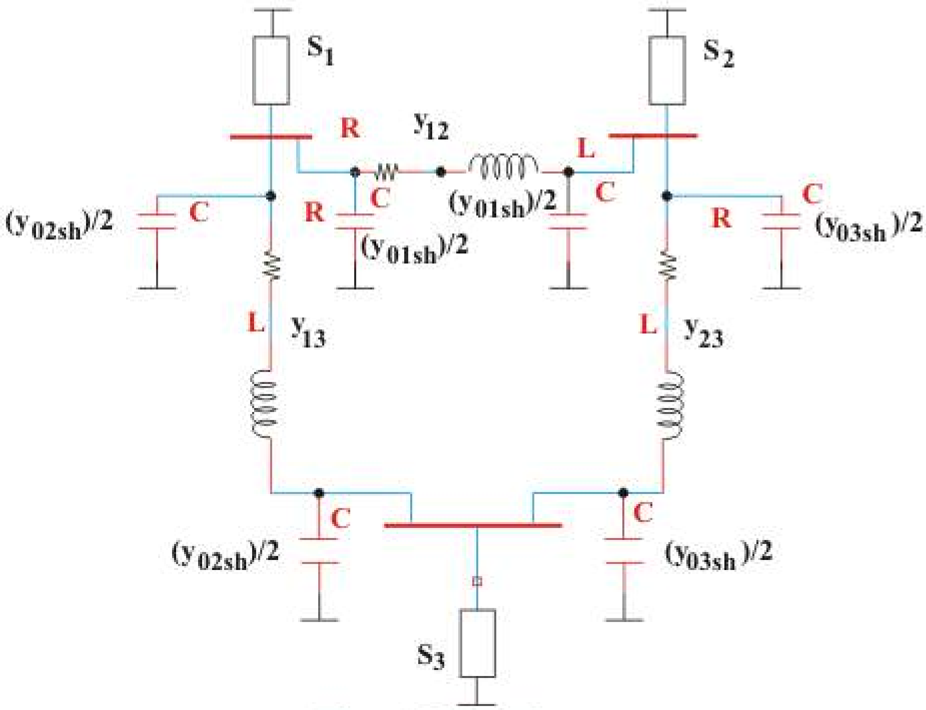


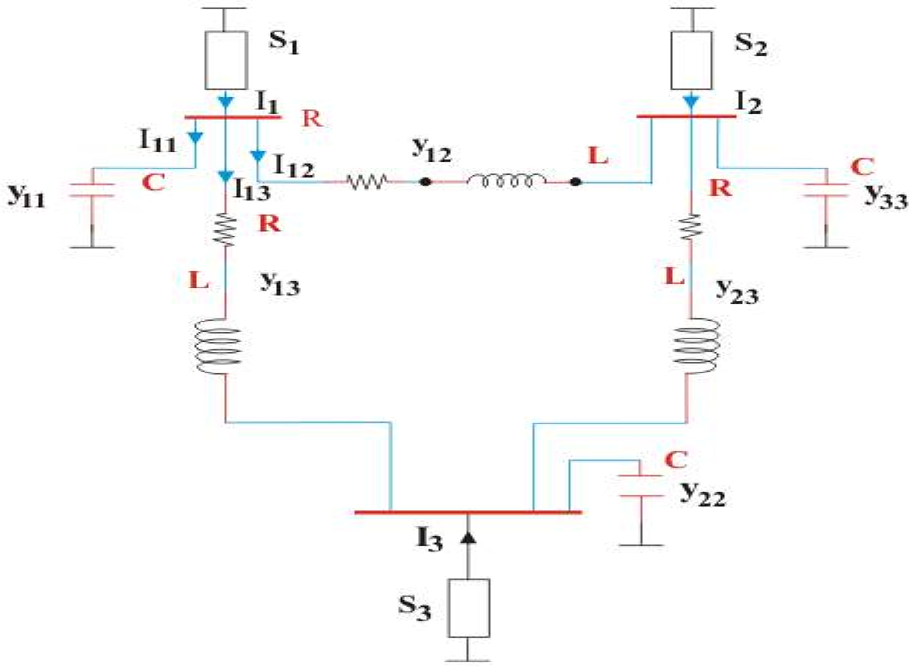
Fig: A 3 Bus System

S_1, S_2, S_3 are net complex power injections into bus 1, 2, 3 respectively
y_{12}, y_{23}, y_{13} are line admittances between lines 1-2, 2-3, 1-3
$y_{01sh}/2$, $y_{02sh}/2$, $y_{03sh}/2$ are half-line charging admittance between lines 1-2, 1-3 and 2-3
The half-line charging admittances connected to the same bus are at same potential and thus can be combined into one

$$y_{11} = \frac{y_{01sh}}{2} + \frac{y_{02sh}}{2}$$

$$y_{22} = \frac{y_{01sh}}{2} + \frac{y_{03sh}}{2}$$

$$y_{33} = \frac{y_{02sh}}{2} + \frac{y_{03sh}}{2}$$



If we apply KCL at bus 1, we have

$$I_1 = I_{11} + I_{12} + I_{13}$$
$$= V_1 y_{11} + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13}$$

Where, V_1, V_2, V_3 are voltage values at bus 1, 2, 3 respectively

$$= V_1(y_{11} + y_{12} + y_{13}) + V_2(-y_{12}) + V_3(-y_{13})$$
$$= V_1 Y_{11} + V_2 Y_{12} + V_3 Y_{13}$$

Where,

$$Y_{11} = y_{11} + y_{12} + y_{13}$$
$$Y_{12} = -y_{12}$$
$$Y_{13} = -y_{13}$$

Similarly by applying KCL at buses 2 and 3 we can derive the values of I_2 and I_3

Finally we have

$$I_1 = V_1Y_{11} + V_2Y_{12} + V_3Y_{13}$$
$$I_2 = V_1Y_{21} + V_2Y_{22} + V_3Y_{23}$$
$$I_3 = V_1Y_{31} + V_2Y_{32} + V_3Y_{33}$$
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$
$$\overline{I}_i = [Y_{BUS}]\overline{V}_i$$

In general for an n bus system

$$I_i = \sum_{j=1}^{n} Y_{ij}V_j$$

Some observations on Y_{BUS} matrix:

1. Y_{BUS} is a sparse matrix
2. Diagonal elements are dominating
3. Off diagonal elements are symmetric
4. The diagonal element of each node is the sum of the admittances connected to it
5. The off diagonal element is negated admittance

Development of Load Flow Equations

The net complex power injection at bus i is given by:

$$S_i = P_i + jQ_i = V_iI_i^* \;.....(1)$$

Taking conjugate

$$S_i^* = P_i - jQ_i = V_i^*I_i \;.....(2)$$

$$I_i = \sum\nolimits_{j=1}^{n} Y_{ij}V_j \;.....(3)$$

Substituting the value of I_i in equation (2)

$$P_i - jQ_i = V_i^*\left[\sum\nolimits_{j=1}^{n} Y_{ij}V_j\right] \;.....(4)$$

To derive the static load flow equation in polar form in equation (4) substitute

$$\overline{V}_i = |V_i|\angle\delta_i$$

$$\overline{V}_j = |V_j|\angle\delta_j$$

$$Y_{ij} = |Y_{ij}|\angle\theta_{ij}$$

On substitution of the above values equation (4) becomes

$$P_i - jQ_i = |V_i|\angle-\delta_i\left[\sum\nolimits_{j=1}^{n} |Y_{ij}|\angle\theta_{ij}|V_j|\angle\delta_j\right] \;.....(5)$$

In equation (5) on multiplication of the terms angles get added. Let's denote δ_i-δ_j by δ_{ij} for convenience

Therefore equation (5) becomes

$$P_i - jQ_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\angle - (\delta_{ij} - \theta_{ij}) \ldots\ldots (6)$$

Expansion of equation (6) into sine and cosine terms gives

$$P_i - jQ_i = \sum_{j=1}^{n} |V_i V_j Y_{ij}| \left[\cos - (\delta_{ij} - \theta_{ij}) - j \ \sin(\delta_{ij} - \theta_{ij})\right]$$

Equating real and imaginary parts we get

$$P_i = \sum_{j=1}^{n} |V_i V_j Y_{ij}| \cos(\delta_{ij} - \theta_{ij}) \ldots\ldots (7)$$

$$Q_i = \sum_{j=1}^{n} |V_i V_j Y_{ij}| \sin(\delta_{ij} - \theta_{ij}) \ldots\ldots (8)$$

Equations (7) and (8) are the static load flow equations in polar form. The above obtained equations are non-linear algebraic equations and can be solved using iterative numerical algorithms.

Similarly to obtain load flow equations in rectangular form in equation (4) substitute

$$\overline{V}_i = e_i + jf_i$$

$$\overline{V}_j = e_j + jf_j$$

$$Y_{ij} = (G_{ij} - jB_{ij})$$

On substituting above values into equation (4) and equating real and imaginary parts we get

$$P_i = \sum_{j=1}^{n} G_{ij}(e_i e_j + f_i f_j) - B_{ij}(f_i e_j - e_i f_j) \ldots\ldots (9)$$

$$Q_i = \sum_{j=1}^{n} G_{ij}(f_i e_j - e_i f_j) + B_{ij}(e_i e_j + f_i f_j) \ldots\ldots (10)$$

Equations (9) and (10) are static load flow equations in rectangular form.