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Subject: Digital Signal Processing EC601

Unit:V

Topic:Filter Designing

- If the no. of samples are odd ie N = odd, then the impulse response would be symmetric about $h\left(\frac{N-1}{2}\right)$

Example :- N = 5

then $h\left(\frac{N-1}{2}\right) = h(2)$

symmetric about h(2).

- If N = ~~even~~ odd, then no. of Multiplier would be $M = \frac{N+1}{2}$.

- If N = even, then response would be symmetric about two values, $h\left(\frac{N-1}{2}\right)$ and $h\frac{(N+1)}{2}$ and no. of Multipliers req. $M = N/2$.

Linear Phase Realisation N is ODD.

$h(n) \neq 0 \qquad 0 \leq n \leq 4.$

$n) = \{h(0), h(1), h(2), h(3), h(4)\}$

$) = h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4}.$

this system be linear system.

$h(0) = h(4) \quad ; \quad h(2) = h(2)$

$h(1) = h(3)$

∴ it can be deduced to

$H(z) = h(0) + h(1) z^{-1} + h_2 z^{-2} + h(1) z^{-3} + h(0) z^{-4}.$

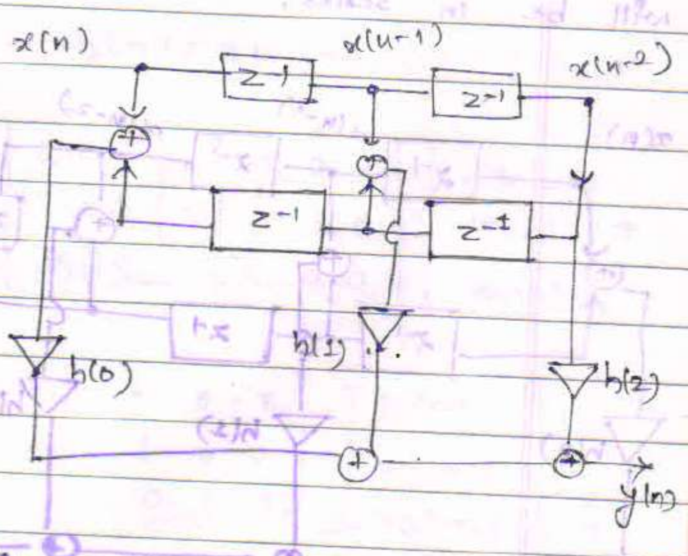
$\Rightarrow \frac{Y(z)}{X(z)} = h(0)[1 + z^{-4}] + h_1[z^{-1} + z^{-3}] + h(2) z^{-2}.$

$\Rightarrow Y(z) = h(0)[1 + z^{-4}]X(z) + h_1[z^{-1} + z^{-3}]X(z) + h(2) z^{-2} X(z)$

$\Rightarrow y(n) = h(0)[x(n) + x(n-4)] + h(1)[x(n-1) + x(n-3)] + h(2) x(n-2).$

- If N = odd then $\frac{N-1}{2}$ elements will be in series.

example -



with This no. of delay elements is reduced from 5 to 3.

for representation of system as computable Matrix, then

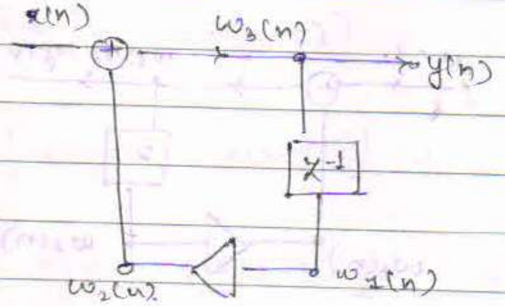
the matrix should be lower Δ matrix,

ie $w_1(n)$ should not depend on $w_1(n)$, $w_2(n)$, $w_3(n)$

$w_2(n)$ shouldnot depend on $w_2(n)$ and $w_3(n)$

$w_3(n)$ should not depend on $w_3(n)$.

Example :-



Here, $w_1(n) = w_3(n-1)$

$w_2(n) = a\,w_2(n)$

$w_3(n) = x(n) + w_2(n)$

$y(n) = w_3(n)$.

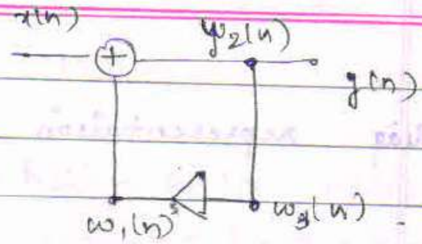
This forms a computational Matrix.

Cond. for Computablity :-

A_c = lower Δ matrix.

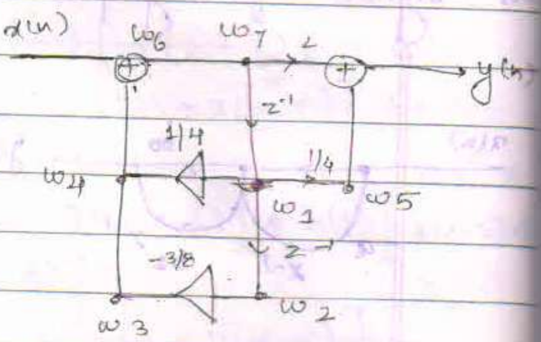
$w(n) = A_c\,w(n) + A_d\,w(n-1) +$
$\beta\,x(n)$.

Example -



Note :- for computablity there has to be a delay element in every loop.

Q.



$w_1 = w_7(n-1)$

$w_2 = w_1(n-1)$

$w_3 = -3/8\,w_2(n)$

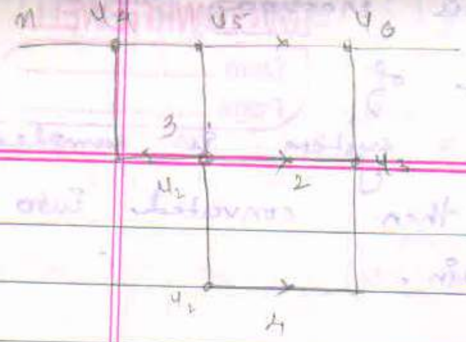
$w_4 = w_3(n) + 1/4\,w_1(n)$

$w_6 = w_4(n) + x(n)$

$w_7(n) = w_6(n)$

$w_5(n) = 1/4\,w_1(n)$

$y(n) = 2w_7(n) + w_5(n)$.



$u_1 = u_5(n-1)$

$u_2 = u_4(n+1)$

$u_3 = 2u_1(n) + 4u_2(n)$

$u_4 = x(n) + 3u_1(n)$

$u_5 = u_4(n)$

$u_6 = u_3(n) + u_5(n)$

$y(n) = u_6(n)$.

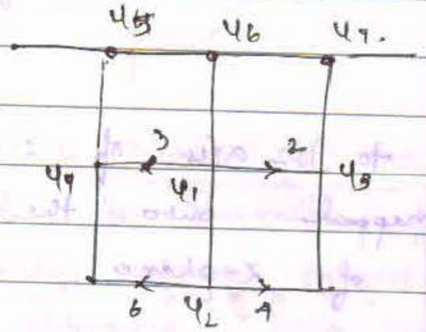
$$\begin{bmatrix} u_1(n) \\ u_2(n) \\ u_3(n) \\ u_4(n) \\ u_5(n) \\ u_6(n) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1(n) \\ u_2(n) \\ u_3(n) \\ u_4(n) \\ u_5(n) \\ u_6(n) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(n-1) \\ u_2(n-1) \\ u_3(n-1) \\ u_4(n-1) \\ u_5(n-1) \\ u_6(n-1) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot x(n)$$

$$y(n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1(n) \\ u_2(n) \\ u_3(n) \\ u_4(n) \\ u_5(n) \\ u_6(n) \end{bmatrix}$$

Q.



→ Designing of Digital System

→ Designing of Digital Filters :-

s domain filter → z- domain filters.

- Digital filters are designed from analog filters -

1) left half s-plane is mapped into the unit circle. of z plane.

2) $-j\Omega$ to $j\Omega$ axis of s-plane is mapped onto the unit circle. of z-plane.

This is the stability criteria while converting filters from s domain to z-domain.

- Designing of IIR & FIR system

IIR → a) Impulse invariant Method.
b) Bilinear transformation,

FIR → Window Method.

- Designing of IIR (filter) System.

1. IMPULSE INVARIENT METHOD

Impulse response of analog signal is made invariant (discrete)

$$H(s) \xrightarrow{ILT} h_a(t) \longrightarrow \underbrace{h(n) = h_a(t)}_{\text{sampled.}}\Big|_{t=nT} \;\Big\downarrow z \;\; H(z)$$

In this Method, Impulse response of analog system is sampled and then converted into z- domain.

Suppose -

$$H_a(s) = \sum_{i=1}^{N} \frac{A_i}{s-P_i} \text{ (All pole filter)}$$

By taking inverse laplace,

$$h_a(t) = ILT[\,H_a(s)\,] = ILT\left[\sum_{i=1}^{N}\right.$$

Since.

$$L^{-1}\left[\frac{1}{s-a}\right] = e^{at}.u(t)$$

$$\therefore\; h_a(t) = \sum_{i=1}^{N} A_i e^{P_i t} u(t) \quad — ⑪$$

Now, sampling the signal!

$$h(n) = h(t)\Big|_{t=nT}$$

$$h(n) = \sum_{i=1}^{N} A_i\, e^{nP_iT} u_0(nT)$$

$$h(n) = \sum_{i=1}^{N} A_i\, e^{nP_iT}.u(n)$$

now taking z-transform -

$$H(z) = ZT[\,h(n)\,] = \sum_{n=0}^{\infty} h(n)\, z$$

$$H(z) = \sum_{n=0}^{\infty} \left[\sum_{i=1}^{N} A_i e^{n p_i T} \right] z^{-n}.$$

$$= \sum_{i=1}^{N} A_i \sum_{n=0}^{\infty} e^{n p_i T} \cdot z^{-n}$$

$$= \sum_{i=1}^{N} A_i \sum_{n=0}^{\infty} \left(e^{p_i T} z^{-1} \right)^{n}.$$

(formula for GP).

$$H(z) = \sum_{i=1}^{N} \frac{A_i}{1 - e^{p_i T} z^{-1}} \quad \text{——} ⅲ$$

Considering eq. ① and ⅲ we have.

from eq. ① pole ⇒ $s = p_i$

from eq. ⅲ pole ⇒ $z = e^{p_i T}$

Since $z = e^{sT}$

$$r e^{j\omega} = e^{(\sigma + j\Omega)T}.$$

$\therefore r = e^{\sigma T}$ {σ = real axis of s-domain}

$\omega = \Omega T$

where T = sampling period

for $r = e^{\sigma T}$

1) when $\sigma > 0$, then $r > 1$ i.e. outside the unit circle.

2) when $\sigma = 0$, then $r = 1$ i.e. circumference of unit circle.

3) when $\sigma < 0$, then $r < 1$ i.e. inside the unit circle.

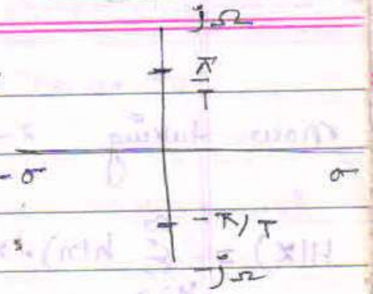
for $\omega = \Omega T$.

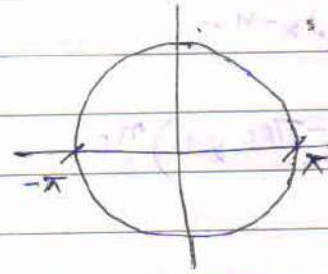
when $\Omega = \pi/4$.

$\omega = \pi$

when $\Omega = -\pi/T$

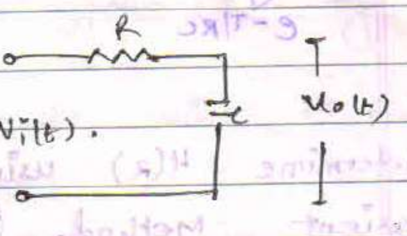
$\omega = -\pi$





This is a <u>Many to one Mapping</u> and is responsible for aliasing effect in IIR and its drawback.

Qo. Convert the given analog filter into digital filter by using impulse invariant method.



T = 1 sec

$$\frac{V_o(s)}{V_i(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1/RC}{s + 1/RC}$$

$$H(s) = \frac{1/RC}{s + 1/RC}$$

$$h_a(t) = \frac{1}{RC} e^{-t/RC} u(t)$$

now by sampling

$$h(n) = \frac{1}{RC} \cdot e^{-\frac{nT}{RC}} \cdot u(n)$$

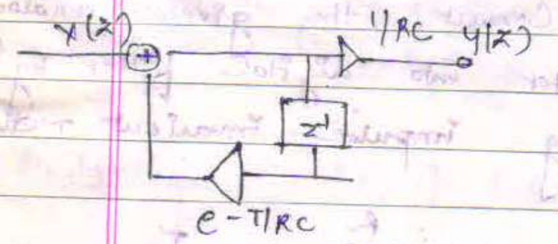
Now taking z-transform.

$$H(z) = \sum_{n=0}^{\infty} h(n) \cdot z^{-n}.$$

$$= \frac{1}{RC} \sum_{n=0}^{\infty} \left(e^{-T/RC} z^{-1} \right)^{n}.$$

$$H(z) = \frac{1/RC}{1 - e^{-T/RC} z^{-1}}$$

On Realisation we have -



Q. Determine H(z) using impulse invarient Method.

$$H(s) = \frac{1}{(s+2)(s+3)} \qquad T = 1 \text{ sec.}$$

$$H(s) = \frac{1}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$$

$1 = A(s+3) + B(s+2)$

at $s = -3$

$1 = B(-1) \Rightarrow B = -1$

at $s = -2$

$1 = A(1) \Rightarrow A = 1$

$$\therefore H(s) = \frac{1}{(s+2)} - \frac{1}{(s+3)}$$

$$\therefore h_a(t) = e^{-2t} \cdot u(t) \cancel{+} - e^{-3t} u(t$$

now by sampling

$$h(n) = e^{-2nT} u(nT) - e^{-3nT} u($$

Taking z-transform.

$$H(z) = \sum_{n=0}^{\infty} h(n) \cdot z^{-n}.$$

$$= \sum_{n=0}^{\infty} \left(e^{-2\cancel{n}T} z^{-1} \right)^{n} + \sum_{n=0}^{\infty} \left(e^{-3T} z^{-}\right.$$

$$= \cancel{e}^{11}$$

$$H(z) = \frac{1}{1 - e^{-2T} z^{-1}} - \frac{1}{1 - e^{-3T} z^{-1}}$$

On Realising we have.

$H(z) \cancel{=} 1 - e$

Qo. Convert analog to digital filter

$$H(s) = \frac{25}{(s+0.1)^2+25}$$

the digital filter must have the resonant freq. of 0.2π make use of IIR Method.

By general eq. of 2nd order system.

$$H(s) = \frac{\Omega^2}{s^2+\Omega^2}$$

on comparing $\Omega^2 = 25$

$\Rightarrow \Omega = 5$.

also given $\omega = 0.2\pi$.

$\therefore$ As we know

$\omega = \Omega T$

$0.2\pi = 5T$

$\therefore T = 0.04\pi$

$= 0.04 \times 3.14$

$T = 0.1256$

As we know.

if $H(s) = \frac{b^2}{(s+a)^2+b^2}$

then $h(t) = e^{-at} \sin bt\, u(t)$

$\therefore h(t) = e^{-0.1t} \sin 5t\, u(t)$

~~$h(n) = e^{-0.1n}$~~

$= (e^{-aT})^n \sin bT.n\, u(n)$

$H(n) = a^n \sin \omega n\, u(n)$.

Now taking z-transform.

$$\therefore H(z) = \frac{a z^{-1} \sin\omega}{1 - 2az^{-1}\cos\omega + a^2 z^{-2}}.$$

On comparing with general eq.

$h(n) = e^{-anT} \sin bnT\, u(n)$

$$H(z) = \frac{e^{-0.1T} z^{-1} \sin 5T}{1 - 2e^{-0.1T} z^{-1} \cos 5T + (e^{-0.1T} z^{-1})^2}$$

$$H(z) = \frac{5e^{-0.012} \sin(0.62) z^{-1}}{1 - 2e^{-0.012} z^{-1} \cos(0.62) + e^{-0.025} z^{-2}}.$$

Qo. Realise given FIR system into

1) cascade using 1st order section.

2) cascade using two 2nd order and one 1st order section

3) Using two 2nd order and 1 third order system.

$H(z) = (1+0.8z^{-1})^5$

①

$$H(s) = \frac{1}{s+1} - \frac{1}{s+2} .$$

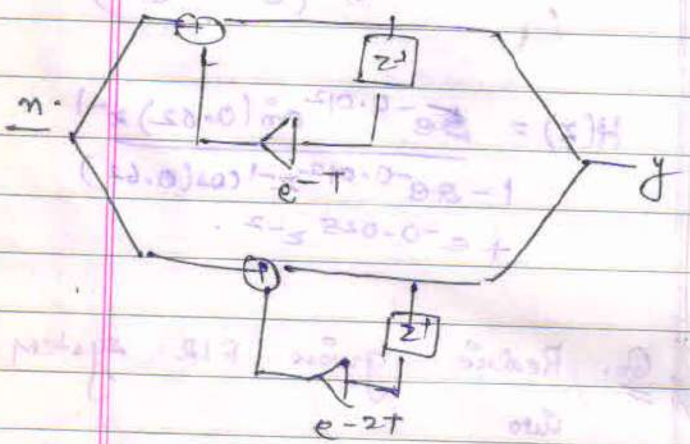
$$h(t) = e^{-t} u(t) - e^{-2t} u(t)$$

$$h(n) = e^{-nT} u(n) - e^{-2nT} u(n)$$

$$H(z) = \sum_{n=0}^{\infty} e^{-nT} z^{-n} - \sum_{n=0}^{\infty} e^{-2nT} z^{-n}.$$

$$= \sum_{n=0}^{\infty} (e^{-T} z^{-1})^{n} - \sum_{n=0}^{\infty} (e^{-2T} z^{-1})^{n}$$

$$= \frac{1}{1 - e^{-T} z^{-1}} - \frac{1}{1 - e^{-2T} z^{-1}}$$



at T=1s,

$$H(z) = \frac{0.2326 z^{-1}}{1 - 0.503 z^{-1} + 0.0498 z^{-2}}$$

②

$$H(s) = \frac{1}{(s+0.5)(s^2+0.5s+2)}$$

$$H(s) = \frac{A}{s+0.2} + \frac{Bs+C}{s^2+0.5s+2}$$

$A = 0.5 \quad B = -0.5, \; C = 0$

$$H(s) = \frac{0.5}{s+0.5} + \frac{-0.5 s}{s^2+0.5s+2}$$

$$= \frac{0.5}{s+0.5} - 0.5 \left[\frac{s+0.25}{(s+0.25)^2 + (1.3919)^2} - \frac{0.25}{(s+0.25)^2 + (1.3919)^2} \right]$$

$$= \frac{0.5}{s+0.5} - 0.5 \left[\frac{s+0.25}{(s+0.25)^2 + (1.3919)^2} \right]$$

$$+ 0.08989 \left[\frac{1.3919}{(s+0.25)^2 + (1.3919)^2} \right]$$

$$H(z) = \frac{0.5}{1 - e^{-0.5T} z^{-1}}$$

$$0.5 \left[\frac{1 - e^{-0.25T} \cos(1.391}{1 - 2e^{-0.25T} \cos(1.391} + e^{-0.5T} z^{-2} \right]$$

then DFT of $x(n)$ is also complex value sequence,

$$X(K) = X_R(K) + j X_I(K)$$

$$0 \leq K \leq (N-1),$$

proof. $\text{DFT}[x(n)] = \sum_{n=0}^{N-1} x(n) W_N^{nk}$

$K = 0, 1, \ldots, (N-1)$

$$= \sum_{n=0}^{N-1} x_R(n) \cos\left(\frac{2\pi}{N}\right).nk - j x_R(n) \sin\left(\frac{2\pi}{N}\right).nk + j x_i(n) \cos\left(\frac{2\pi}{N}\right)nk + x_i(n) \sin\left(\frac{2\pi}{N}\right).nk \Big].$$

$$= \sum_{n=0}^{(N-1)} \left[x_R(n) \cos\left(\frac{2\pi}{N}\right)nk + x_i(n) \sin\left(\frac{2\pi}{N}\right).nk\right] + j\left[x_i(n) \cos\left(\frac{2\pi}{N}\right)nk - x_R(n) \sin\left(\frac{2\pi}{N}\right)nk\right]$$

$$X_I(K) = \sum_{n=0}^{(N-1)} x_i(n) \cos\left(\frac{2\pi}{N}\right)nk - x_R(n) \sin\left(\frac{2\pi}{N}\right)nk.$$

Now, IDFT of the sequence.

$$x(n) = \frac{1}{N} \sum_{k=0}^{(N-1)} X(K). W_N^{-nk}.$$

$$x(n) = \frac{1}{N} \sum_{K=0}^{(N-1)} X(K) \left[\cos\left(\frac{2\pi nk}{N}\right) \quad j \sin\left(\frac{2\pi nk}{N}\right)\right]$$

$$x_R(n) = \frac{1}{N} \sum_{k=0}^{N-1} x_R(K). \cos\left(\frac{2\pi nK}{N}\right) - X_I \sin\left(\frac{2\pi nK}{N}\right)$$

$$x_I(n) = \frac{1}{N} \sum_{K=0}^{N-1} X_I(K) \cos\left(\frac{2\pi nK}{N}\right) + X_R(K) \sin\left(\frac{2\pi}{N}\right) n.k.$$

Case I :- when $x(n)$ is purely real.

If $x(n)$ is even.

$$x(n) = x((-n))_N$$
$$= x(N-n).$$

If $x(n)$ is odd,

$$x(n) = -x((-n))_N$$
$$= -x(N-n).$$

$x(n) = x_R(n)$ $\quad x_i(n) = 0$

$$X_R(n) = \sum_{n=0}^{N-1} x_R(n) \cos\left(\frac{2\pi}{N} nk\right).$$

$K = 0, 1, \ldots, (N-1)$

$$X_I(K) = \sum_{n=0}^{(N-1)} -X_R(n) \sin\left(\frac{2\pi nk}{N}\right)$$

$K = 0, 1, \ldots, (N-1)$

If $x(n)$ is real valued sequence - $x(n) = x^*(n)$

BILINEAR TRANSFORMATION

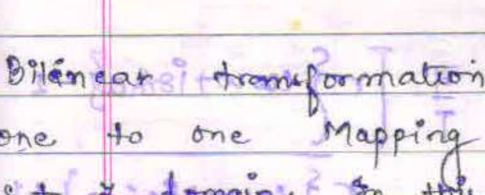
Impulse invarient method is not used for high pass filters & band reject filters due to freq. aliasing prob.

This method is used for LPF and band limitted filters, where cut off freq. is very low.

For high pass filter & Band reject filters alternative tech. is used that is k/a "Bilinear Transformation".

Bilinear transformation is one to one Mapping from s to z domain. In this Method we are using trapezoidal rule of numerical integration. This is also k/a Tustin transformation.

Trapezoidal Rule :-
(for discrete integration)



$$\int_a^b f(t)\,dt = \frac{(b-a)}{2}\left[f(a)+f(b)\right]$$

procedure for Bilinear

Suppose Transfer fn is

$$H(s) = \frac{b}{s+a}$$

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a}.$$

$$sY(s) + aY(s) = b\cdot X(s).$$

$$sY(s) = bX(s) - aY(s)$$

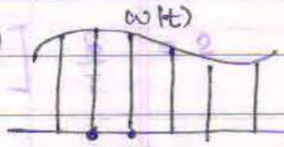
let $bX(s) - aY(s) = W(s)$

then.

$$sY(s) = W(s) \quad — ①$$

Taking inverse laplace.

$$\frac{dy(t)}{dt} = w(t) \quad — ⑪$$



T= sampling period.

integrating eq. ⑪ b/w two consecutive samples (n-1)T and nT, we get -

$$\int_{(n-1)T}^{nT} \frac{dy}{dt}\cdot dt = \int_{(n-1)T}^{nT} w(t)\cdot dt$$

$$\left[y(t)\right]_{(n-1)T}^{nT} = \frac{T}{2}\left[w(nT) + w(n-1)T\right]$$

by trapezoidal rule.

$$\therefore y(nT) - y(n-1)T = \frac{T}{2}\left[w(nT) + w(n-1)T\right]$$

Shows sampling done

usual diff eq. would be

$$y(n) - y(n-1) = \frac{T}{2}\left[w(n) + w(n-1)\right] \quad — ⑪⑪$$

Taking z-transform of eq. (III)

$$Y(z) - z^{-1}Y(z) = \frac{T}{2}\left[W(z) + z^{-1}W(z)\right]$$

$$Y(z)\left[1 - z^{-1}\right] = \frac{T}{2}\left[1 + z^{-1}\right]W(z)$$

$$\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]Y(z) = W(z) \text{ —— (IV)}$$

from eq. ① and eq. (IV)

$$sY(s) = W(s)$$

$$\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]Y(z) = W(z).$$

on comparing two equations we get -

$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] \text{ —— (V)}$$

Thus with this we can determine the transfer fun of a filter given in s domain by :-

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

It is k/a bilinear 'mation as linear eq. in both num & denominator.

This is one to one transformation

Properties of Bilinear Transformation

Since

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

Here $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

put $s = \sigma + j\Omega$ & $z = re^{j\omega}$

$$\sigma + j\Omega = \frac{2}{T}\left[\frac{1-(re^{j\omega})^{-1}}{1+(re^{j\omega})^{-1}}\right]$$

$$\sigma + j\Omega = \frac{2}{T}\left[\frac{1-(r\cos\omega + ir\sin\omega)^{-1}}{1+(r\cos\omega - ir\sin\omega)^{-1}}\right]$$

$$\sigma + j\omega = \frac{2}{T}\left[\frac{r\{\cos\omega + i\sin\omega\} - 1}{r\{\cos\omega + i\sin\omega\} + 1}\right]$$

$$(\sigma + j\omega)\left[(r\cos\omega + ri\sin\omega) + 1\right]T = 2\{r[\cos\omega + i\sin\omega] - 1\}$$

$$\sigma + j\Omega = \frac{2}{T}\left[\frac{1 - \frac{1}{re^{j\omega}}}{1 + \frac{1}{re^{j\omega}}}\right]$$

$$= \frac{2}{T}\left[\frac{re^{j\omega} - 1}{re^{j\omega} + 1}\right]$$

$$= \frac{2}{T}\left[\frac{re^{j\omega} - 1}{re^{j\omega} + 1}\right] \times \left[\frac{re^{-j\omega} + 1}{re^{-j\omega} + 1}\right]$$

$$= \frac{2}{T}\left[\frac{(re^{j\omega}-1)(re^{-j\omega}+1)}{(re^{j\omega}+1)(re^{-j\omega}+1)}\right]$$

$$= \frac{2}{T}\left[\frac{r^2+re^{j\omega}-re^{-j\omega}-1}{r^2+re^{j\omega}+re^{-j\omega}+1}\right]$$

$$\sigma+j\Omega = \frac{2}{T}\left[\frac{r^2-1+r(2j\sin\omega)}{r^2+r(2\cos\omega)+1}\right]$$

$$\sigma+j\Omega = \frac{2}{T}\left[\frac{r^2-1}{r^2+r(2\cos\omega)+1} + \frac{r(2j\sin\omega)}{r^2+r(2\cos\omega)+1}\right]$$

$$\therefore\ \sigma = \frac{2}{T}\left[\frac{r^2-1}{r^2+r(2\cos\omega)+1}\right]$$

$$\sigma = \frac{2}{T}\left[\frac{2r\sin\omega}{r^2+2r\cos\omega+1}\right]$$

mapping -

① for σ

when $\sigma > 0$ ie $r > 1$ (outside circle)

when $\sigma = 0$ ie $r = 1$ (circumference)

when $\sigma < 0$ ie $r < 1$ (inside circle)

② for Ω

for stability let $r = 1$.

$$\Omega = \frac{2}{T}\left[\frac{2\sin\omega}{2+2\cos\omega}\right]$$

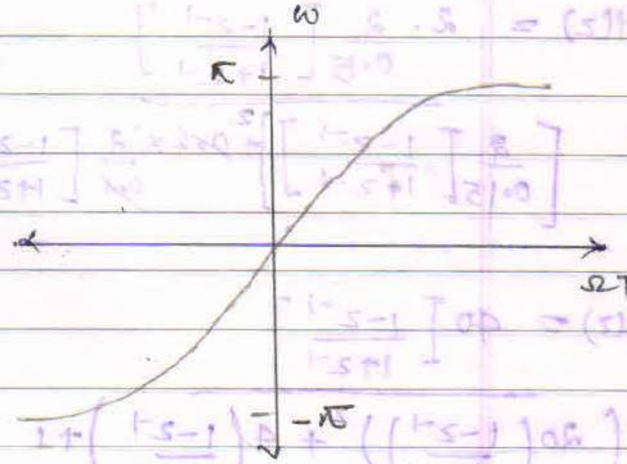
$$\Omega = \frac{2}{T}\left[\frac{\sin\omega}{1+\cos\omega}\right]$$

$$\Omega = \frac{2}{T}\left[\frac{2\sin\omega/2\ \cos\omega/2}{2\cos^2\omega/2}\right]$$

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

$$\frac{\omega}{2} = \tan^{-1}\left(\frac{\Omega T}{2}\right)$$

$$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$$



Note :- This non-linearity of freq. causes freq. wrapping effect in BLT (similar to companding. This is major drawback of BLT.

Q.1 Convert the analog filter with system fn.

$$H(s) = \left[\frac{s+0.1}{(s+0.1)^2+16}\right]$$

into digital IIR filter by using BLT with resonant freq. $\omega_0 = \pi/2$.

$$H(z) = \frac{4(1-z^{-2}) + 0.1(1+z^{-1})^2}{(4.1-3.9z^{-1})^2 + 16(1+z^{-1})^2}$$

Q: Apply bilinear transformation

$$H(s) = \frac{2s}{s^2+0.2s+1}$$

with T = 0.15 second & find H(z).

② $H(s) = \frac{2s}{s^2+0.2s+1}$

put $s = \frac{2}{0.15}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$$H(z) = \frac{2\cdot\frac{2}{0.15}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}{\left[\frac{2}{0.15}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]\right]^2 + 0.2\times\frac{2}{0.1}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+1}$$

$\frac{2}{0.15}$ $\frac{400}{15}$ $\frac{200}{15}$ 3

$$H(z) = \frac{40\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}{\left(20\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2 + 4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1}$$

$$H(z) = \frac{40\left(\frac{1-z^{-1}}{1+z^{-1}}\right]}{400\left(\frac{1+z^{-2}-2z^{-1}}{(1+z^{-1})^2}\right) + \frac{4(1-z^{-1})}{(1+z^{-1})}+1}$$

$$H(z) = \frac{\left[\frac{40(1-z^{-1})}{(1+\cancel{z^{-1}})}\right]}{\frac{400(1-z^{-2}+2z^{-1}) + 4(1-z^{-1})(1+z^{-1}) + (1+z^{-1})^2}{(1+z^{-1})^2}}$$

$$H(z) = \frac{40(1-z^{-1})(1+z^{-1})}{400-400z^{-2}+800z^{-1} + 4(1-z^{-2})+(1+z^{-2}+2z^{-1})}$$

$$= \frac{40(1-z^{-1})(1+z^{-1})}{400-400z^{-2}+800z^{-1}+4-4z^{-2}+1+z^{-2}+2z^{-1}}$$

$$= \frac{40(1-z^{-2})}{-403z^{-2}+802z^{-1}+405}$$

−1600
24
1624
+ 9
1615

1600
24
1624
9
1633

$$H(s) = \frac{2s}{s^2+0.2s+1}$$

$$= \frac{2\left[\frac{40}{3}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]}{\left[\frac{40}{3}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + \frac{2}{10}\left(\frac{40}{3}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right) + 1}$$

3200
18
3182

$$= \frac{\frac{80}{3}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}{\frac{1600}{9}\frac{(1+z^{-2}-2z^{-1})}{(1+z^{-1})^2} + \frac{8}{3}\frac{(1-z^{-2})}{(1+z^{-1})^2} + (1+z^{-1})^2}$$

$$= \frac{\frac{80}{\cancel{3}}\left[\frac{1-z^{-1}}{1+\cancel{z^{-1}}}\right]}{\frac{1600(1+z^{-2}-2z^{-1})+24(1-z^{-2})+9(1+z^{-1})^2}{3\cancel{9}(1+z^{-1})^2}}$$

$$= \frac{80\times3(1-z^{-2})}{1600(1+z^{-2}-2z^{-1})+24(1-z^{-2})+9(1+z^{-2}+2z^{-1})}$$

$$H(z) = \frac{240(1-z^{-2})}{1600 + 1600z^{-2} - 3200z^{-1} + 24 - 24z^{-2} + 9 + 9z^{-2} + 18z^{-1}}$$

$$H(z) = \frac{240(1-z^{-2})}{585z^{-2} - 1633 - \cancel{+1615}z^{-2} - 3182z^{-1}}$$

① $H(s) = \left[\frac{s+0.1}{(s+0.1)^2+16}\right]$

$H(s) = \cancel{\omega}^2 \frac{s+9}{(s+a)^2+\omega^2}$

$\Rightarrow \Omega^2 = 16 \Rightarrow \Omega = 4$

$\therefore \omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$

$\pi/2 = 2\tan^{-1}\left(\frac{4 \cdot T}{2}\right)$

$T = 1/2$

put $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$= \frac{4}{\cancel{8}}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$$H(z) = \frac{\frac{4(1-z^{-1})}{(1+z^{-1})} + 0.1}{\left[\frac{4(1-z^{-1})}{(1+z^{-1})} + 0.1\right]^2 + 16}$$

$$= \frac{\frac{4(1-z^{-1}) + 0.1(1+z^{-1})}{\cancel{(1+z^{-1})}}}{\frac{16(1-z^{-1})^2 + 0.01(1+z^{-1})^2 + 0.8(1-z^{-2}) + 16(1+z^{-1})^2}{(1-z^{-1})^{\cancel{2}}}}$$

$$= \frac{4 - 4z^{-1} + 0.1 + 0.1z^{-1}}{}$$

$$= \frac{\left[4(1-z^{-1}) + 0.1(1+z^{-1})\right] \times (1-z^{-1})}{\left[16(1+z^{-2}-2z^{-1}) + 0.01(1+z^{-2}+2z^{-1}) + 0.8 - 0.8z^{-2} + 16(1+z^{-2}-2z^{-1})\right]}$$

$$= \frac{0.1(1+z^{-1})^2 + 4(1-z^{-2})}{(4.1-3.9z^{-1})^2 + 16(1+z^{-1})^2}$$