Properties of DTFS

Q2 find the DTFs and sketch the amp and phase spectra.

Q2 // $x(n) = 5 + \sin\left(\frac{n\pi}{2}\right) + \cos\left(\frac{n\pi}{4}\right)$

$N_1 = \frac{2\pi}{\omega_0} m = \left(\frac{2\pi}{\pi/2}\right) \times m = 4$ for $m=1$

$N_2 = \left(\frac{2\pi}{\omega_0}\right) \times m = 8$ $\quad \frac{N_1}{N_2} = \frac{4}{8} = \frac{1}{2}$

$2N_1 = N_2 = 8$.

$x(n) = 5 + \sin\left(\frac{n\pi}{2}\right) + \cos\frac{n\pi}{4}$

$= 5 + \left[\frac{e^{\frac{j\pi n}{2}} - e^{-\frac{j\pi n}{2}}}{2j}\right] + \frac{e^{jn\pi/4} + e^{-jn\pi/4}}{2}$ ——— (1)

$x(n) = \sum_{k=-3}^{4} C_k e^{j2\pi kn/N}$ $\quad = 8/4$ ——— (2) $= \sum C_n e^{jn\pi kn/4}$

$N = 8$

on comparing eqn (1) and eqn (2)

$= C_{-3} e^{-j3\pi n/4} + C_{-2} e^{-j\pi n/2} + C_{-1} e^{-j\pi n/4} + C_0 + C_1 e^{j\pi n/4}$

$+ C_2 e^{j\pi n/2} + C_3 e^{j3\pi n/4} + C_4 e^{j\pi n}$ ——— (3)

$= \left[C_{-2} = -\frac{1}{2j} = \frac{-1\times j}{2\times j^2} = \frac{-j}{-2} = j^{(0.5)} \quad C_2 = \frac{1}{2j}\right.$

$\left. C_0 = 5 ; \quad C_1 = 1/2, \quad C_{-1} = 1/2\right.$

$|C_k| = \frac{1}{2}$, $k = -1, -2, 1, 2$ $\quad \sqrt{0.7(0.5)^2} = 0.5$

$= 5, \; k = 0$

$= 0, \quad k = -3, 3, 4, -4$.

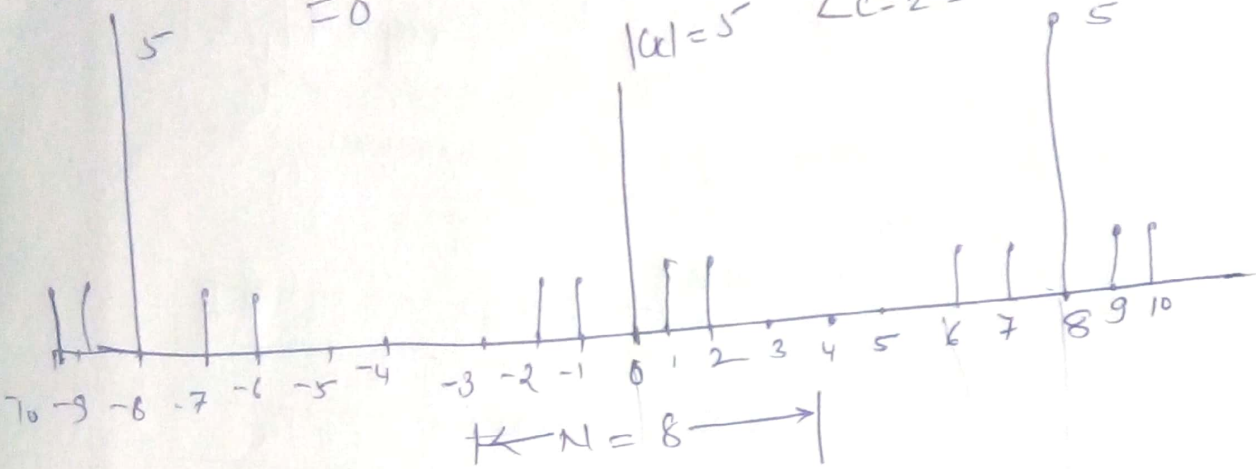
$\angle C_k = \frac{\pi}{2}$, for $k = 2$

$|C_{-2}| = \frac{-1}{2j} = \frac{-j}{2} = 0.5j$

$\angle C_{-2} = \pi/2$, $k = -2$ $\quad \angle C_{-2} = \tan^{-1}\left(\frac{0.5}{0}\right) = \infty = \pi/2$

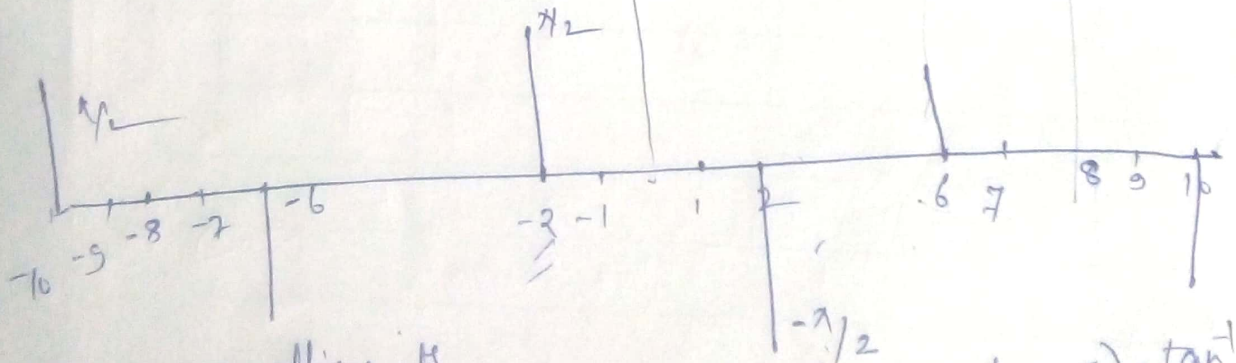
$= \pi/2$ for $k = 2$ $\quad |C_2| = \frac{1}{2j} = \frac{j}{2j^2} = -0.5j$

$= 0$ $\quad \angle C_{-2} = -\pi/2$.



Magnitude spe.

Phase spectrum



Mag. coefficients

$|C_k|$ $\quad C_2 = \frac{1}{2j} \quad \frac{1 \times j}{2j \times j} = \frac{j}{2} = -0.5j$

$|C_2| = 0 - 0.5j = \sqrt{0 + 0.5^2} = 0.5$

$|C_{-2}| = \frac{-1}{2j} = 0 + 0.5j = 0.5$

$|C_1| = 1/2 \pm 0j \quad |C_{-1}| = 1/2$

$\pi/2 = \tan^{-1}\left(\frac{0.5}{0}\right) = \tan^{-1}(\pi$

$\angle C_{-2} = \tan^{-1}(\pi/2$

Real term

$\angle C_1 = \frac{0}{0.5} = \tan^{-1}(0)$

$\angle C_{-1} = 0$

$= \tan^{-1}$

Q. 3 $x(n) = \cos\left(\frac{6\pi}{13}n + \frac{\pi}{6}\right)$

$N = \frac{2\pi \times 13 \times m}{6\pi 3}$

$= m = 3, \quad N = 13$ samples

Synthesis $x(n) = \sum_{k=N} x(k) e^{jk\omega_0 n}$

given eqn $= \frac{e^{j\left(\frac{6\pi}{13}n + \pi/6\right)} + e^{-j\left(\frac{6\pi}{13}n + \pi/6\right)}}{2}$

$= \frac{1}{2} e^{j\frac{6\pi}{13}n} \cdot e^{j\pi/6} + \frac{1}{2} e^{-j\pi/6} \cdot e^{-j\frac{6\pi}{13}n}$

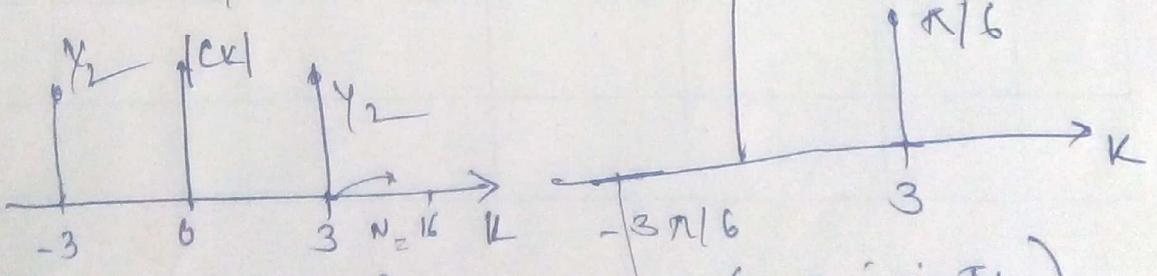
$= \frac{1}{2} e^{j\pi/6} e^{jk\left(\frac{2\pi}{13}\right)n} + \frac{1}{2} e^{-j\pi/6} e^{(j)(-3)\left(\frac{2\pi}{13}\right)n}$

$k = 3 \qquad k = -3$

$x(n) = \sum_{k=N} X(k) e^{jk\omega_0 n}$

$X(3) = \frac{1}{2} e^{j\pi/6} \rightarrow \theta = \pi/6$

$X(-3) = \frac{1}{2} e^{-j\pi/6} \quad \theta = -\pi/6$



$X(3) = \frac{1}{2}\left(\cos\frac{\pi}{6} + j\sin\frac{\pi}{6}\right)$

$X(-3) = \frac{1}{2}\left(\cos\frac{\pi}{6} - j\sin\pi/6\right)$

$|X_3| = \sqrt{\left(\frac{1}{2}\right)^2\cos^2\frac{\pi}{6} + \left(\frac{1}{2}\right)^2\sin^2\left(\frac{\pi}{6}\right)}$

$= \frac{1}{2}$ ✓

$= Y_2$

or $\tan^{-1}\left(\frac{1}{2}\sin\pi/6\right)$

$\tan^{-1}(\tan\pi/6) \quad Y_2\cos\pi/6$