Kawar

UNIT-4 (Part. I DTFS DFS.

Fourier Analysis of discrete time signal (DTFS, DTFT)

A discrete time signal is periodic

$$N = \left(\frac{2\pi}{\omega_0}\right) \times m = \left(\frac{2\pi}{\omega_0}\right) \times 4$$

or $\omega_0 = \frac{2\pi}{N}$, K = index of fourier series

The exponential fourier series consist

$e^{j\left(\frac{2\pi}{N}\right)n}$ is periodic with period N.

for any range of N of K

$\omega = \frac{2\pi}{N}$

$$\boxed{x(n) = \sum_{K=0}^{N-1} X_K e^{j\omega_0 n k} = \sum_{K=K_0}^{K_0+N-1} X_K e^{j2\pi k n/N} \quad n = 0, \ldots N-1}$$

Synthesis eqn

where $n \rightarrow$ time instants

where X_K = Fourier series coefficients

where $X_K = C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$, $k = 0, 1, \ldots N-1$

Analysis

for any range of N of n

K - represents different exponentials

for continuous signal

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}$$

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt$$

5(2)

As spectra is similar for $\omega = 0, 2\pi$

Consider the signal $x(n) = \sin \omega_0 n$. is periodic, $\frac{2\pi}{\omega_0} = N$. (integer)

or $N = \frac{2\pi}{\omega_0}$ or $= \frac{2\pi}{2\pi} =$

Expanding the signal in exponential form.

$$x(n) = \frac{1}{2j}\left[e^{j(2\pi/N)n} - e^{-j(2\pi/N)n}\right]$$

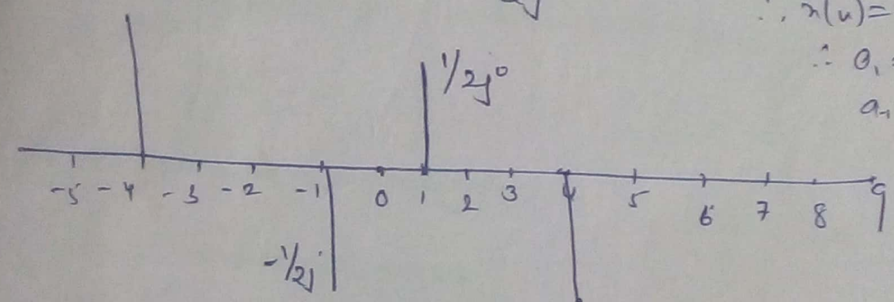
As periodic signal it will repeat

$a_1 = \frac{1}{2j}$, $a_{-1} = \frac{-1}{2j}$, Now if $N=5$, $\omega_0 = \frac{2\pi}{5}$

$\therefore x(n) = \sin\left(\frac{2\pi}{5}\right)n$

$\therefore a_1 = a_6 = a_{11}$

$a_{-1} = a_{-6} = a_{-11}$.



Q)

$x(n) = \cos\left(\frac{\pi}{4} n\right)$ $N=8$

$N = \frac{2\pi}{\omega_0} = \frac{2\pi(m)}{\pi/4} = 8$ for $(m=1)$

$$x(n) = \frac{1}{2}\left[e^{j(\frac{\pi}{4})n} + e^{-j(\pi/4)n}\right] \quad \text{—(1)}$$

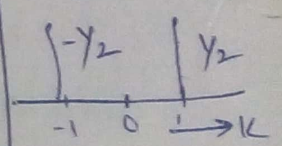
$\because N=8$, $x(n) = \sum_{k=-3}^{4} C_k e^{j2\pi kn/N}$

$$= \sum_{k=-3}^{4} e^{j\pi kn/4} \quad \text{——— (2)}$$

$$x(n) = \frac{1}{2}\sum e^{j\frac{2\pi 1}{8}} + \frac{1}{2}\sum e^{j\frac{2\pi(-1)}{8}}$$

$C_k = 1/2$ ($k=1$), $1/2$ ($k=-1$)

mag spec



no phase.

real.

$\tan^{-1}\left(\frac{0}{0.5}\right) = 0$