



“Lecture notes on”
UNIT- 4
FRICTION
(BT-204) (ENGINEERING MECHANICS)

(Exclusively for A-1 (CS) & A-4 (IT))

By

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INTRODUCTION TO FRICTION

FRICTION

Whenever the surfaces of two bodies are in contact, there is some resistance to sliding between them. The opposing force to the movement is called friction or force of friction. It is due to interlocking of surfaces as a result of the presence of some roughness and irregularities at the contact surfaces. The resisting force acts in the direction opposite to the movement. A force of friction comes into play whenever there is a relative motion between two parts. Some energy is wasted in order to overcome the resistance due to force of friction.

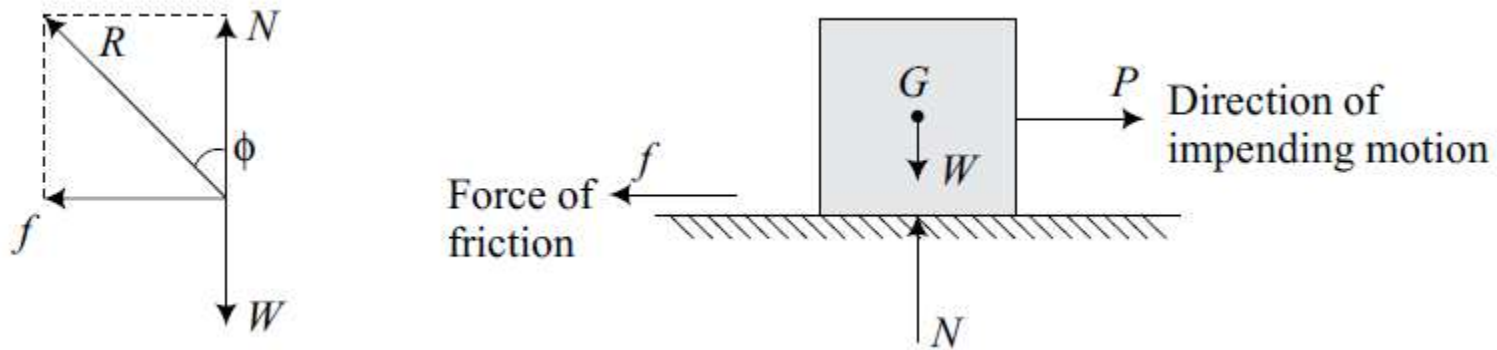


Fig.1 Force of friction

INTRODUCTION TO FRICTION

FRICTION

Force of friction or frictional force is the opposing force to the movement of one body over the surface of another body.

In Fig.1

W = Weight of the body (mg)

N = Normal reaction

f = Friction force

P = Force applied to the body

R = Total reaction

ϕ = Angle of friction

INTRODUCTION TO FRICTION

IMPORTANT DEFINITIONS

(a) *Angle of Friction*

It is the angle made by the resultant (R) of the normal reaction (N) and limiting force of friction (f) and made with the direction of normal reaction.

R is the resultant of normal reaction N and force of friction f .

$$R = \sqrt{N^2 + f^2}$$

ϕ is the angle of friction

$$\tan \phi = \frac{f}{N}$$

or

$$\phi = \tan^{-1} \frac{f}{N}$$

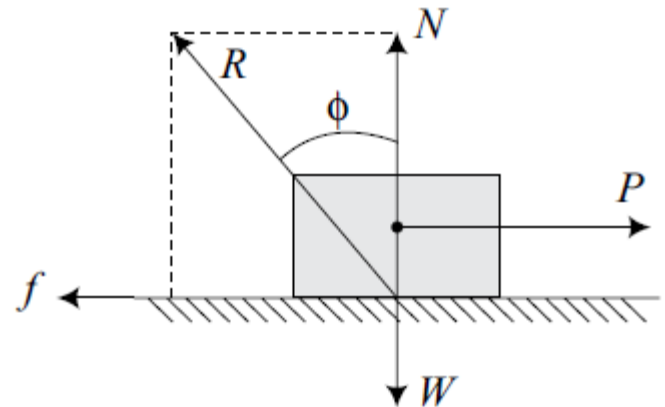


Fig.2.Angle of Friction

INTRODUCTION TO FRICTION

(b) Coefficient of friction

It is ratio of limiting frictional force and the normal reaction.

The coefficient of friction,

$$\mu = \frac{f}{N} = \tan \phi$$

\therefore

$$f = \mu N$$

μ depends upon the nature of contacting surface. Its value is very low for lubricated surfaces and high for dry friction.

INTRODUCTION TO FRICTION

(c) *Angle of Repose*

A body of weight W is lying on a rough plane inclined at an α with the horizontal. The body is in equilibrium under the action of following forces:

- (i) *Weight of the body W* . It has two components: $W \sin \alpha$ parallel to inclined plane and $W \cos \alpha$ normal to the plane.
- (ii) *Normal reaction, N* acting in a direction normal to inclined plane.
- (iii) *Friction force, f* acting in a direction opposite to the motion.

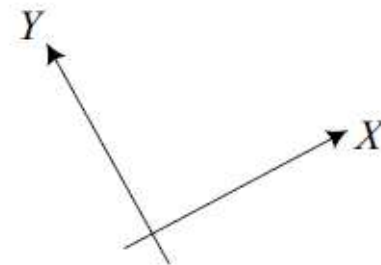
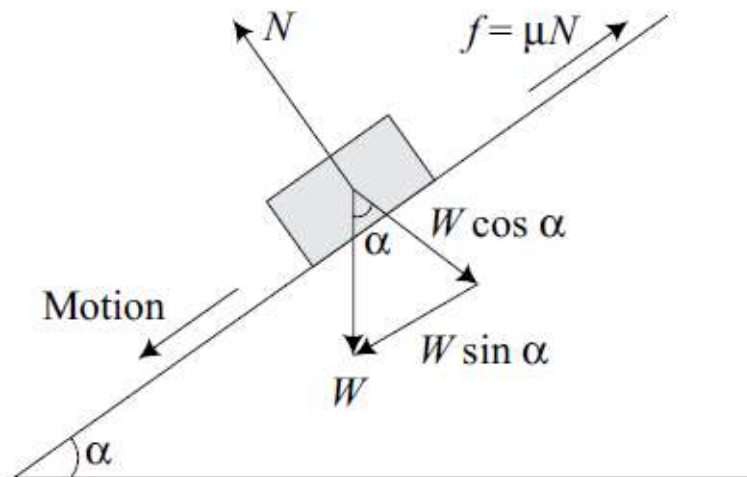


Fig.3.Angle of Repose

INTRODUCTION TO FRICTION

When the body tends to slide down the plane, the frictional force must act up the plane and when the body is being pulled up, the force of friction acts downwards to oppose the motion.

Selecting the reference coordinate system with X -axis in the direction of inclined plane and Y -axis perpendicular to inclined plane,

Applying equilibrium conditions,

$$\Sigma F_X = 0$$

$$f = W \sin \alpha$$

$$\Sigma F_Y = 0$$

$$N = W \cos \alpha$$

$$\therefore \frac{f}{N} = \frac{W \sin \alpha}{W \cos \alpha} = \tan \alpha$$

But,
$$\frac{f}{N} = \mu = \tan \phi$$

where ϕ is called the angle of friction.

resolved into rectilinear components. The unknown forces are found from the equations of equilibrium.

INTRODUCTION TO FRICTION

Rough Horizontal Plane

There can be following cases of motion of body on rough horizontal plane. The equilibrium conditions for each case are discussed.

(a) No moving force

$$P = 0$$

∴ Friction force,

$$f = 0$$

$$W = N$$

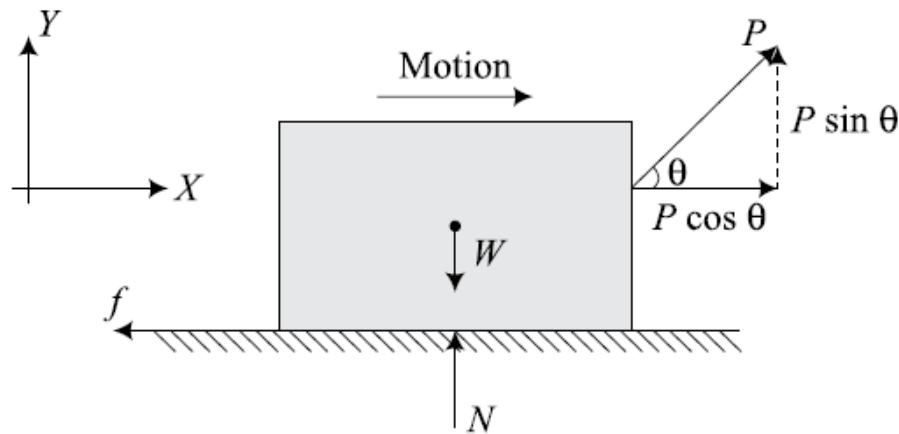


Fig.4. Rough Horizontal Plane

INTRODUCTION TO FRICTION

Rough Horizontal Plane

(b) Body moving under pull or push

A force P is applied to the body. The force body diagram is shown.
Considering the equilibrium of the body.

$$\Sigma F_X = 0$$

$$P \cos \theta = f$$

But by definition, $f = \mu N$

$$\therefore P \cos \theta = \mu N \quad \dots(1)$$

$$\Sigma F_Y = 0$$

$$\therefore N = W \pm P \sin \theta$$

$$\therefore W = N + P \sin \theta$$

$$\text{or } N = W - P \sin \theta \quad \dots(2)$$

From equations (1) and (2)

$$P \cos \theta = \mu(W - P \sin \theta)$$

$$P \cos \theta + \mu P \sin \theta = \mu W$$

$$P \left(\cos \theta + \frac{\sin \phi}{\cos \phi} \sin \theta \right) = \frac{\sin \phi}{\cos \phi} W$$

$$\left[\because \mu = \tan \phi = \frac{\sin \phi}{\cos \phi} \right]$$

INTRODUCTION TO FRICTION

Rough Horizontal Plane

$$p(\cos \theta \cos \phi + \sin \theta \sin \phi) = W \sin \phi$$

$$P \cos (\theta - \phi) = W \sin \phi$$

$$P = \frac{W \sin \phi}{\cos (\theta - \phi)}$$

For P to be minimum, $\cos (\theta - \phi)$ should be maximum,

$$\therefore \cos (\theta - \phi) = 1$$

$$\theta = \phi$$

The angle of inclination of force P should be equal to the angle of friction, ϕ .

INTRODUCTION TO FRICTION

Rough Horizontal Plane

Q.1. The force required to pull the body of weight 50 N on a rough horizontal surface is 20 N where it is applied at an angle of 25° with the horizontal as shown. Determine the coefficient of friction and magnitude of reaction N between the body and the horizontal surface. Does the reaction pass through the centre of gravity of the body?

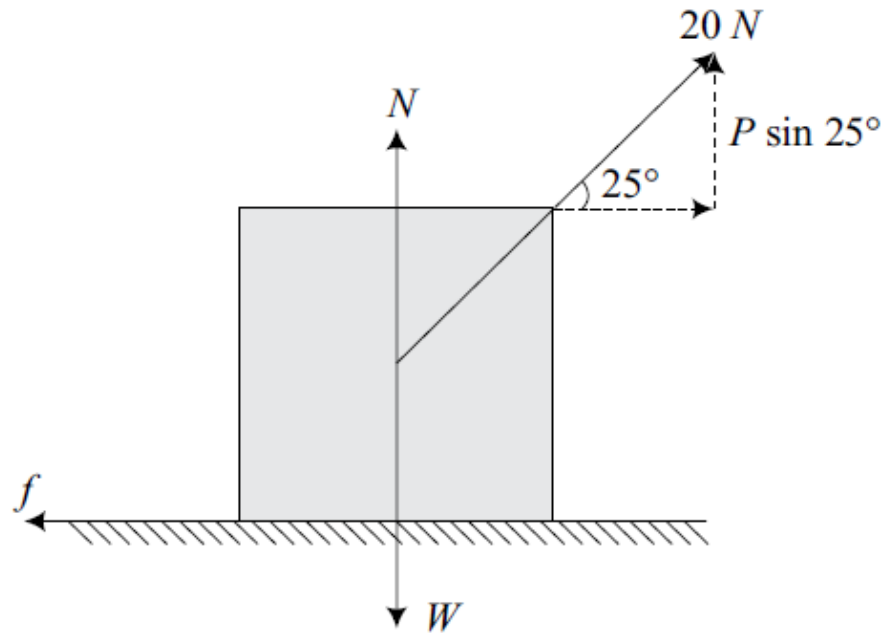


Fig.4.1 Rough Horizontal Plane

INTRODUCTION TO FRICTION

Rough Horizontal Plane

Solution: The body is in equilibrium

$$\begin{aligned}\therefore \quad \Sigma F_y &= 0 \\ N &= W - P \sin 25^\circ \\ &= 50 - 20 \times 0.42 \\ &= 41.55 \text{ N}\end{aligned}$$

$$\begin{aligned}\therefore \quad \Sigma F_x &= 0 \\ f &= 20 \cos 25^\circ = 18.13 \text{ N}\end{aligned}$$

Now,
$$\mu = \frac{f}{N} = \frac{18.13}{41.55} = \mathbf{0.436} \quad \mathbf{Ans.}$$

The reaction passes through the centre of gravity of the body as it is equal and opposite to weight of body W .

INTRODUCTION TO FRICTION

Rough Horizontal Plane

Q.2. A block of weight 5 kN is pulled by a force P as shown. The coefficient of friction between the contact surface is 0.35. Find the direction θ for which P is minimum and find the corresponding value of P .

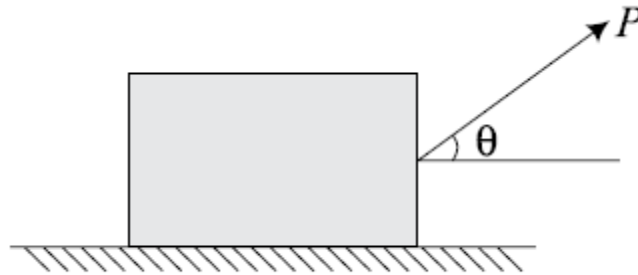
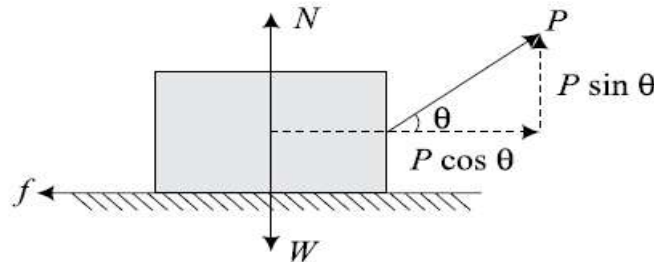


Fig.4.2 Rough Horizontal Plane

Solution:

- (i) Draw free body diagram of the block.
- (ii) Apply equilibrium conditions.



INTRODUCTION TO FRICTION

Rough Horizontal Plane

$$\Sigma F_x = 0$$

$$f = P \cos \theta$$

$$\Sigma F_y = 0$$

$$W = N + P \sin \theta$$

(iii) The angle of inclination of force P will be equal to the angle of friction for minimum value of P .

$$\begin{aligned}\therefore \theta &= \phi = \tan^{-1} \mu = \tan^{-1} 0.35 \\ &= 20.48^\circ\end{aligned}$$

$$P = \frac{W \sin \phi}{\cos (\theta - \phi)}$$

For P to be minimum, $\cos (\theta - \phi) = 1$

$$\therefore P = W \sin \phi = 5 \sin 20.48^\circ = \mathbf{1.75 \text{ kN} \quad \text{Ans.}}$$

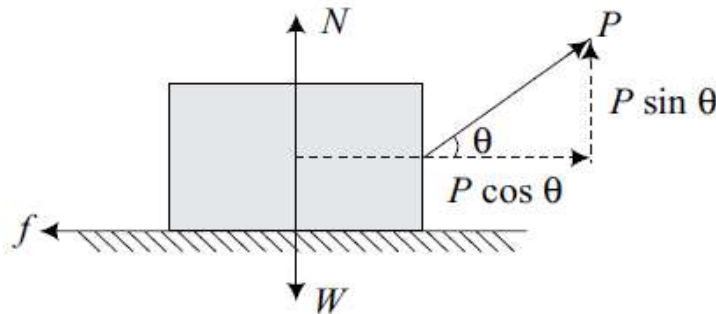
INTRODUCTION TO FRICTION

Rough Horizontal Plane

Q.3. Obtain the expression for minimum force required to drag a body on a rough horizontal plane.

Solution:

- (i) Draw free body diagram.
- (ii) Apply equilibrium conditions.



$$\Sigma F_x = 0$$

$$\therefore f = P \cos \theta = \mu N \quad \dots(1)$$

$$\Sigma F_y = 0$$

$$N = W + P \sin \theta \quad \dots(2)$$

INTRODUCTION TO FRICTION

Rough Horizontal Plane

From equations (1) and (2)

$$\begin{aligned}P \cos \theta &= \mu N \\&= \mu(W - P \sin \theta)\end{aligned}$$

$$P \cos \theta + \mu P \sin \theta = \mu W$$

Now,
$$\mu = \tan \phi = \frac{\sin \phi}{\cos \phi}$$

$$P \left(\cos \theta + \frac{\sin \phi}{\cos \phi} \sin \theta \right) = \frac{\sin \phi}{\cos \phi} \cdot W$$

$$P(\cos \theta \cos \phi + \sin \theta \sin \phi) = W \sin \phi$$

$$P \cos (\theta - \phi) = W \sin \phi$$

$$P = \frac{W \sin \phi}{\cos (\theta - \phi)}$$

For P to be minimum, $\cos (\theta - \phi)$ should be maximum

$$\therefore \cos (\theta - \phi) = 1$$

$$\theta = \phi$$

$$\text{Minimum value of } P = W \sin \phi$$

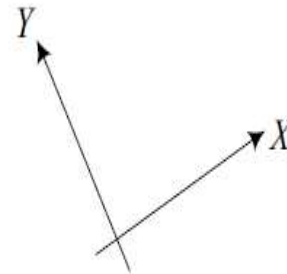
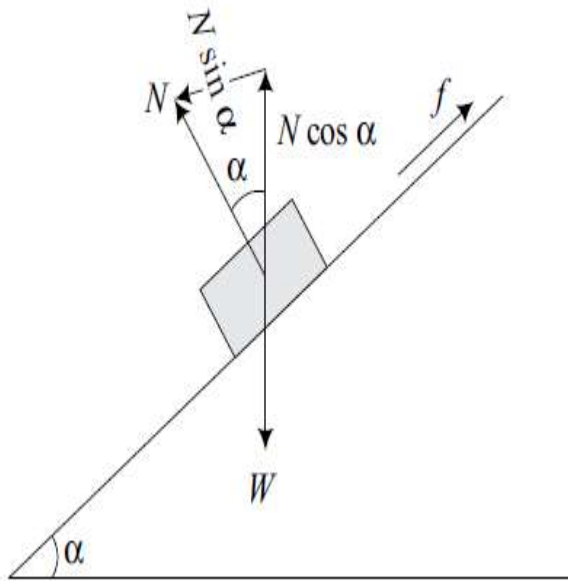
$$\text{or} \quad = W \sin \theta \quad \text{Ans.}$$

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

(a) *Equilibrium Condition for Different Angle of Inclination*

- (i) Angle of inclination less than angle of friction
1. Draw free body diagram.
 2. Apply equilibrium conditions.



$$\Sigma F_x = 0$$

$$f = \mu N = N \sin \alpha$$

$$\Sigma F_y = 0$$

$$W = N \cos \alpha$$

Fig.5 Rough Inclined plane ($\alpha < \phi$)

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

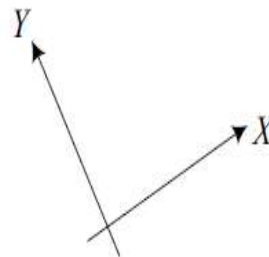
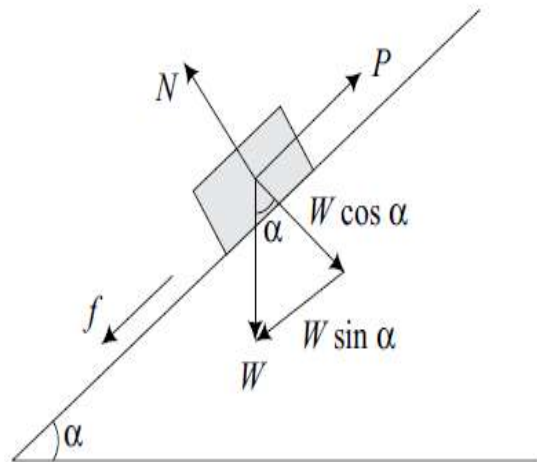
The body is in equilibrium.

(i) Angle of inclination more than angle of friction.

The body will slide down and an upward force P is required to restrict the body from moving down. The restricting force can be applied in different ways.

1. *Along the inclined plane*

Draw free body diagram.



Equilibrium conditions.

$$\Sigma F_x = 0$$

$$f = P - W \sin \alpha$$

$$\Sigma F_y = 0$$

$$N = W \cos \alpha$$

Fig.6 Force parallel to inclined plane

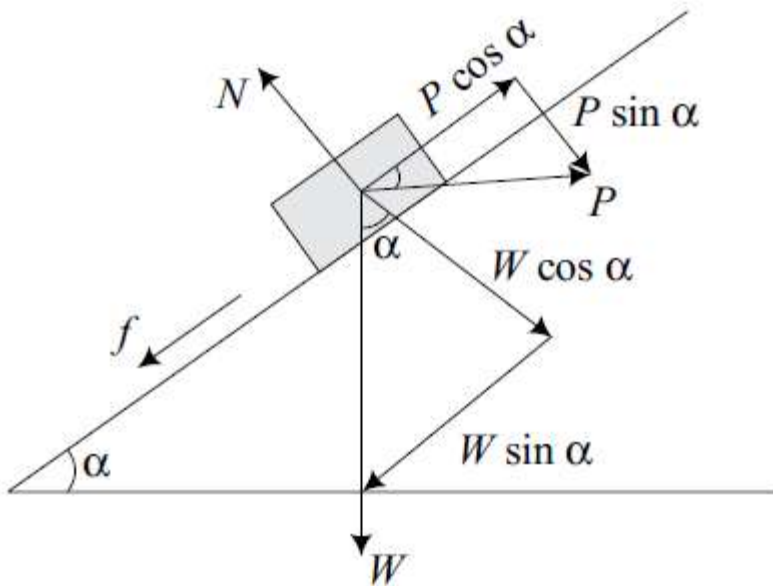
INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

2. *The restricting force applied horizontally*

Draw free body diagram.

Apply equilibrium condition.



$$\Sigma F_x = 0$$

$$f = P \cos \alpha - W \sin \alpha$$

$$\Sigma F_y = 0$$

$$N = W \cos \alpha + P \sin \alpha$$

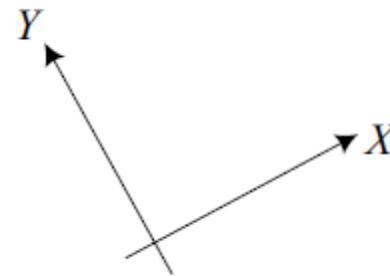


Fig.7 Force applied horizontally

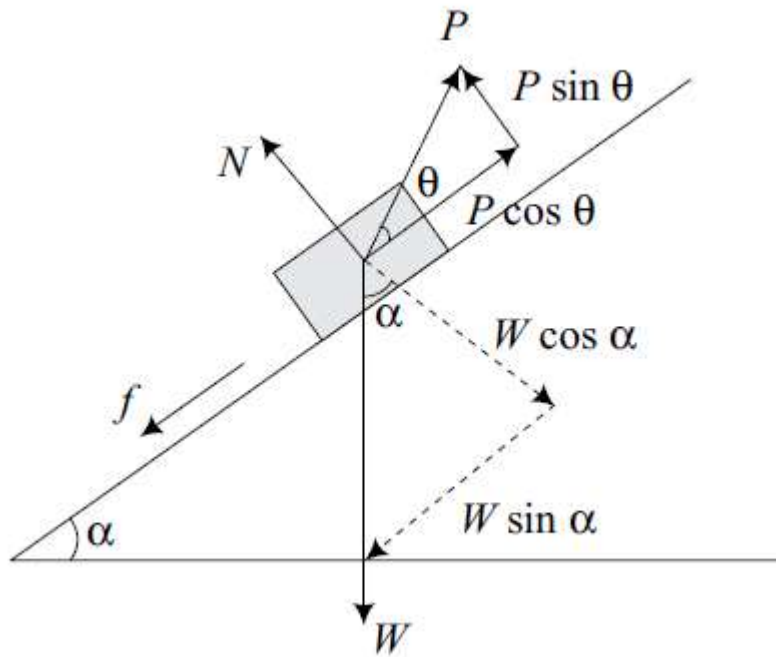
INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

3. Force applied at an angle θ with the inclined plane

Draw free body diagram.

Apply equilibrium conditions.



$$\Sigma F_x = 0$$

$$f = P \cos \theta - W \sin \alpha$$

$$\Sigma F_y = 0$$

$$N = W \cos \alpha - P \sin \theta$$

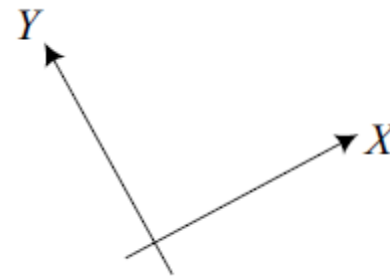


Fig.8 Force applied at an angle θ with plane

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

(b) Body sliding downwards

A body of weight W is on the verge of sliding downwards. A minimum force P is required to restrict the motion.

- (i) Draw free body diagram.
- (ii) Apply equilibrium conditions.

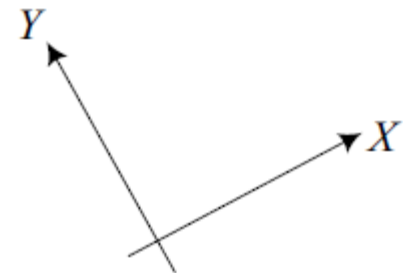
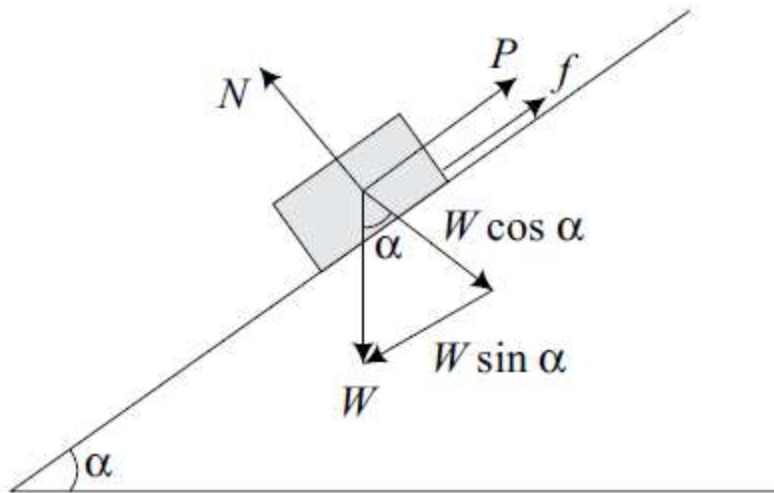


Fig.9 Body sliding downwards

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

(b) Body sliding downwards

$$\Sigma F_x = 0$$

$$f = \mu N = W \sin \alpha - P$$

$$\therefore P = W \sin \alpha - \mu N \quad \dots(1)$$

$$\Sigma F_y = 0$$

$$N = W \cos \alpha \quad \dots(2)$$

From equations (1) and (2)

$$P = W \sin \alpha - \mu (W \cos \alpha)$$

$$= W(\sin \alpha - \mu \cos \alpha)$$

$$= W \left(\sin \alpha - \frac{\sin \phi}{\sin \theta} \cos \alpha \right)$$

$$\therefore \mu = \tan \phi = \frac{\sin \phi}{\cos \phi}$$

$$P \cos \phi = W(\sin \alpha \cos \phi - \cos \alpha \cos \phi)$$

$$\therefore P = \frac{W \sin(\alpha - \phi)}{\cos \phi}$$

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

(c) Body upwards an inclined plane

Minimum force required to keep the body in equilibrium.

- (i) Draw free body diagram.
- (ii) Apply equilibrium conditions.

$$\Sigma F_x = 0$$

$$f = P - W \sin \alpha$$

$$\therefore P = W \sin \alpha + f = W \sin \alpha + \mu N \quad \dots(1)$$

$$\Sigma F_y = 0$$

$$N = W \cos \alpha \quad \dots(2)$$

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

(c) Body upwards an inclined plane

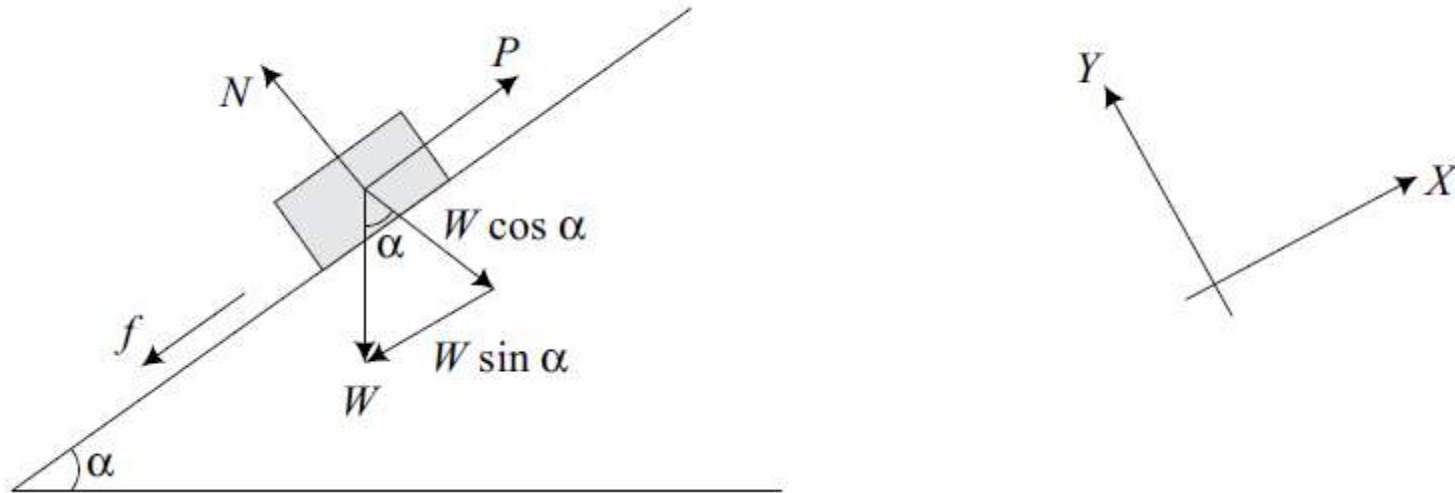


Fig.10 Body upwards an inclined plane

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

(c) Body upwards an inclined plane

From equations (1) and (2)

$$P = W \sin \alpha + \mu W \cos \alpha$$

$$= W \sin \alpha + \frac{\sin \phi}{\cos \phi} W \cos \alpha \quad \left[\mu = \tan \phi = \frac{\sin \phi}{\cos \phi} \right]$$

$$P \cos \phi = W \sin \alpha \cos \phi + W \sin \cos \alpha$$

$$= W \sin (\alpha + \phi)$$

\therefore

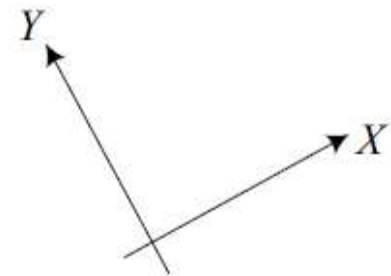
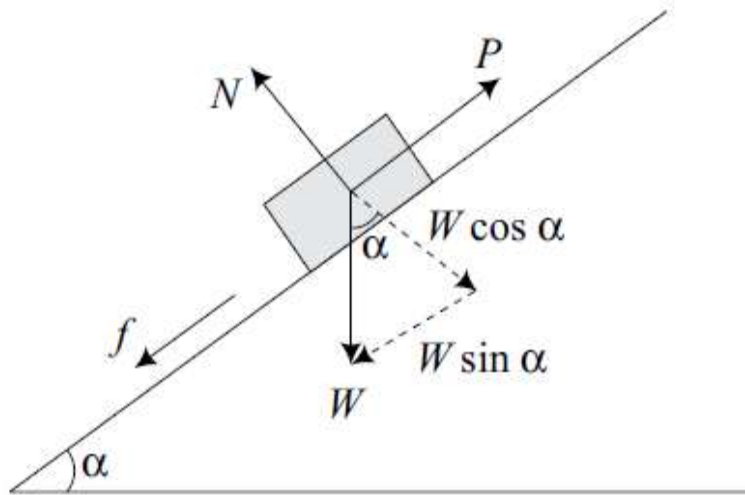
$$P = \frac{W \sin (\alpha + \phi)}{\cos \phi}$$

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

Q.4. A body of weight 500 N is pulled up along an inclined plane having an inclination of 30° with the horizontal. If the coefficient of friction between the body and the plane is 0.3 and the force is applied parallel to inclined plane, determine the force required.

Solution:



INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

$$W = 500 \text{ N}$$

$$\alpha = 30^\circ$$

$$\mu = 0.3$$

Angle of friction,

$$\phi = \tan^{-1} \mu = \tan^{-1} 0.3 = 16.7^\circ$$

The force of friction will be acting downwards as the body is tending to move upwards.

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

Draw free body diagram and apply equilibrium conditions.

$$\Sigma F_x = 0$$

$$P - W \sin \alpha = f = \mu N$$

$$\therefore P = W \sin \alpha + \mu N \quad \dots(1)$$

$$\Sigma F_y = 0$$

$$N = W \cos \alpha \quad \dots(2)$$

From equations (1) and (2)

$$\begin{aligned} P &= W \sin \alpha + \mu W \cos \alpha \\ &= 500 \sin 30^\circ + 0.3 \times 500 \cos 30^\circ \\ &= 250 \times 0.5 + 0.3 \times 500 \times 0.866 \\ &= 250 + 129.9 = \mathbf{379.9 \text{ N}} \quad \mathbf{Ans.} \end{aligned}$$

INTRODUCTION TO FRICTION

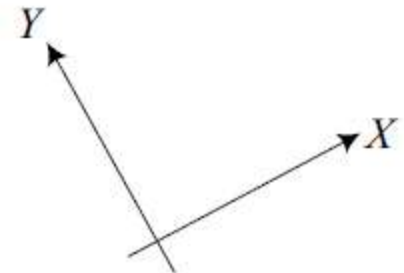
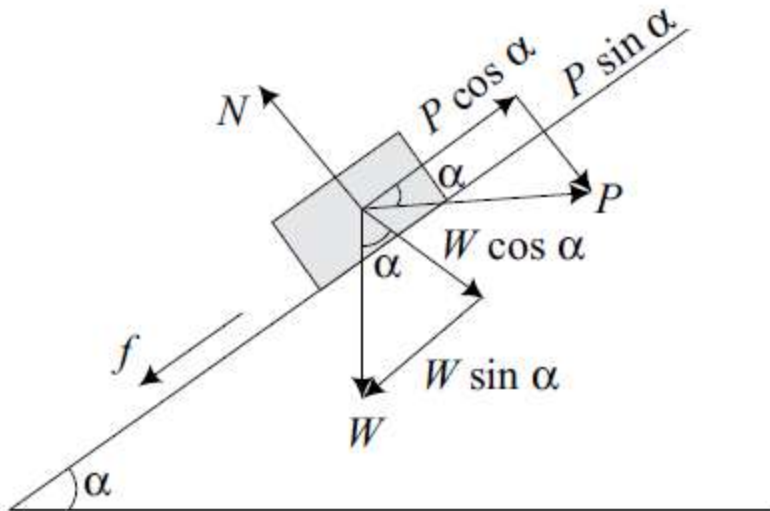
ROUGH INCLINED PLANE

Q.5. A body of weight W is placed on a rough inclined plane having inclination α to the horizontal. The force P is applied horizontal to drag the body. If the body is on the point of motion up the plane, prove that P is given by:

$$P = W \tan (\alpha + \phi)$$

where, ϕ = angle of friction.

Solution: Draw free body diagram and apply equilibrium conditions.



INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

$$\Sigma F_x = 0$$

$$P \cos \alpha = W \sin \alpha + f = W \sin \alpha + \mu N \quad \dots(1)$$

$$\Sigma F_y = 0$$

$$N = W \cos \alpha + P \sin \alpha \quad \dots(2)$$

From equations (1) and (2)

$$P \cos \alpha = W \sin \alpha + \mu(W \cos \alpha + P \sin \alpha)$$

$$= W \sin \alpha + \mu W \cos \alpha + \mu P \sin \alpha$$

$$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

$$\therefore P = W \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}$$

But, $\mu = \tan \phi$ (By definition)

$$\begin{aligned}\therefore P &= W \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha} \\ &= W \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi} \\ &= W \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}\end{aligned}$$

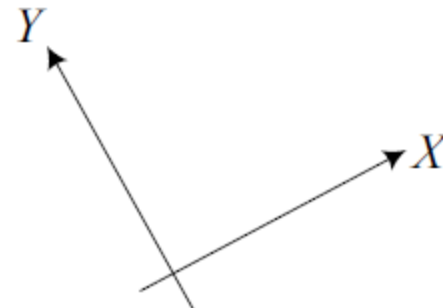
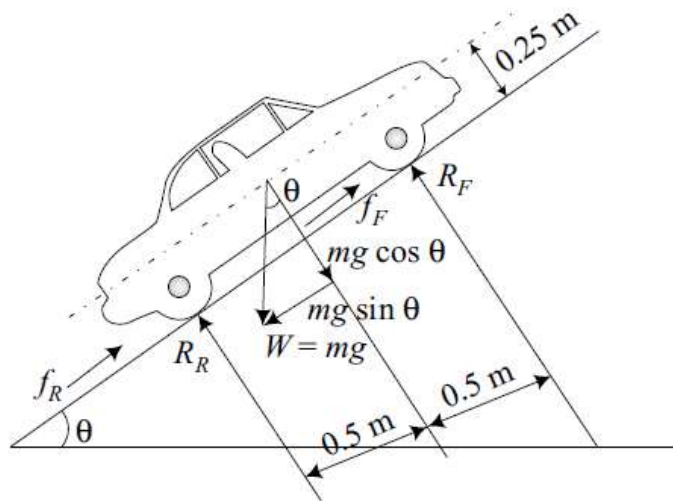
$$\therefore P = W \tan (\alpha + \phi) \quad \text{Proved.}$$

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

Q.6. A four wheel drive car as shown has a mass of 2000 kg with passengers. The roadway is inclined at an angle θ with the horizontal. If the coefficient of friction between tyres and road is 0.3, what is the maximum inclination θ that can be climbed?

Solution: Draw free body diagram as shown. The wheels are rotating clockwise to climb the roadway, the forces of friction on the front wheel and rear wheel f_F and f_R will be acting upwards to oppose the direction of motion of the wheel.



INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

Apply equilibrium conditions,

$$\Sigma F_x = 0$$

$$\begin{aligned}\therefore f_R + f_F &= mg \sin \theta \\ &= 2000 + 9.81 \sin \theta \\ &= 19620 \sin \theta \text{ (N)}\end{aligned}$$

$$\Sigma F_y = 0$$

$$\begin{aligned}R_R + R_F &= mg \cos \theta \\ &= 2000 \times 9.81 \cos \theta \\ &= 19620 \cos \theta \text{ (N)}\end{aligned}$$

INTRODUCTION TO FRICTION

ROUGH INCLINED PLANE

$$\text{But, } f_R = \mu R_R$$

$$\text{and } f_F = \mu R_F$$

$$\therefore f_R + f_F = \mu(R_R + R_F)$$

$$19620 \sin \theta = \mu(R_R + R_F)$$

$$\therefore R_F + R_R = \frac{19620 \sin \theta}{0.3} = 65400 \sin \theta$$

$$= 19620 \cos \theta$$

$$\therefore \frac{\sin \theta}{\cos \theta} = \frac{19620}{65400} = 0.3$$

$$\therefore \theta = \mathbf{16.7^\circ}$$

INTRODUCTION TO FRICTION

LADDER FRICTION

A ladder is placed against a rough wall and a rough floor with coefficient of friction μ_2 and μ_1 respectively.

Example: A ladder, AB of length L and of weight W is placed against a rough wall and rough floor. The reactions R_A and R_B will be acting perpendicular to the point of supports at floor and wall. The ladder has tendency to slide down. Therefore, the forces of friction f_A and f_B will be acting to oppose the motion.

Draw free body diagram of ladder and apply conditions of equilibrium.

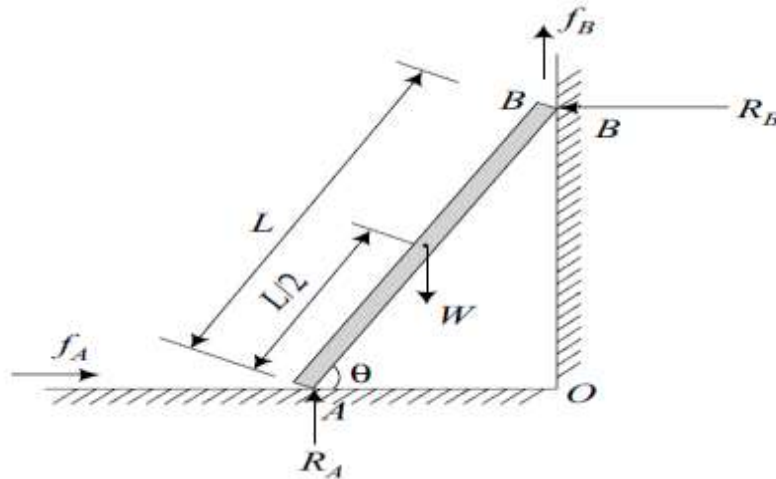


Fig.11 Ladder friction

INTRODUCTION TO FRICTION

LADDER FRICTION

$$\Sigma F_x = 0 \quad \therefore f_A = R_B$$

$$\Sigma F_y = 0 \quad W = R_A + f_B$$

The number of unknowns are four. Therefore, take moments about point O .

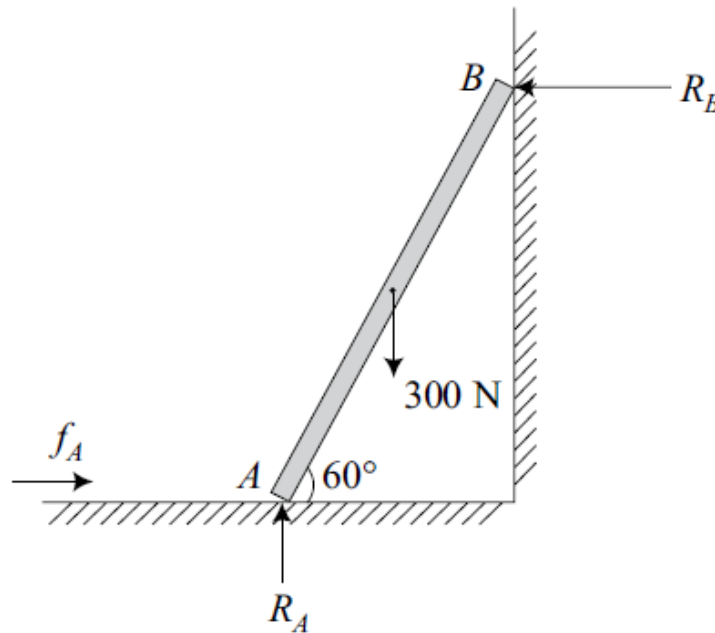
$$\Sigma M_O = 0$$

INTRODUCTION TO FRICTION

LADDER FRICTION

Q.7. A uniform ladder of weight 300 N rests against a smooth vertical wall a rough horizontal floor making an angle 60° with the horizontal. Find the force of friction at floor.

Solution: Draw free body diagram of ladder AB and apply condition of equilibrium.



INTRODUCTION TO FRICTION

LADDER FRICTION

As the wall is smooth, $f_B = 0$.

$$\Sigma F_x = 0$$

$$\therefore f_A = R_B$$

$$\mu R_A = R_B$$

$$\Sigma F_y = 0$$

$$R_A = W = 300 \text{ N}$$

$$\therefore f_A = \mu R_A = 300\mu$$

Assume $\mu = 0.3$ between ladder and floor.

The force of friction,

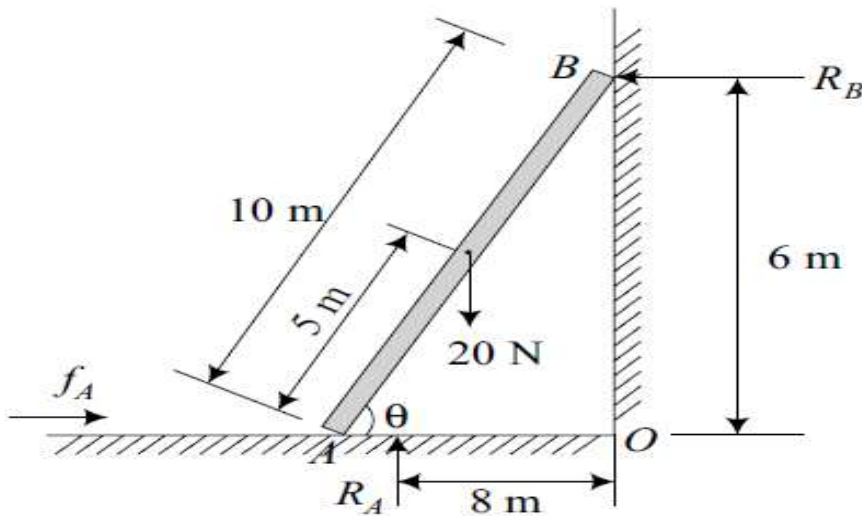
$$f_A = 0.3 \times 300 = \mathbf{90 \text{ N}} \quad \mathbf{Ans.}$$

INTRODUCTION TO FRICTION

LADDER FRICTION

- Q.8.** A uniform ladder of length 10 m and weighing 20 N is placed against a smooth vertical wall with its lower end 8 m from the wall. In this position the ladder is just to slip. Determine:
- Coefficient of friction between ladder and floor.
 - Frictional force acting on the ladder at the point of contact between the ladder and floor.

Solution: Draw free body diagram of ladder.



INTRODUCTION TO FRICTION

LADDER FRICTION

Angle of inclination,

$$\cos \theta = \frac{8}{10} = 0.8$$

$$\therefore \theta = 36.87^\circ$$

$$\sin \theta = 0.6$$

Applying equilibrium conditions,

$$\Sigma F_x = 0$$

$$\therefore f_A = R_B$$

$$\Sigma F_y = 0$$

$$R_A = W = 20 \text{ N} \quad (\because f_B = 0)$$

$$\Sigma M_O = 0$$

$$R_A \times 8 = 20 \times 4 + R_B \times 6$$

$$R_B = \frac{R_A \times 8 - 20 \times 4}{6} = \frac{20 \times 8 - 20 \times 4}{8}$$

$$= 10 \text{ N}$$

Force of friction, $f_A = R_B = 10 \text{ N}$

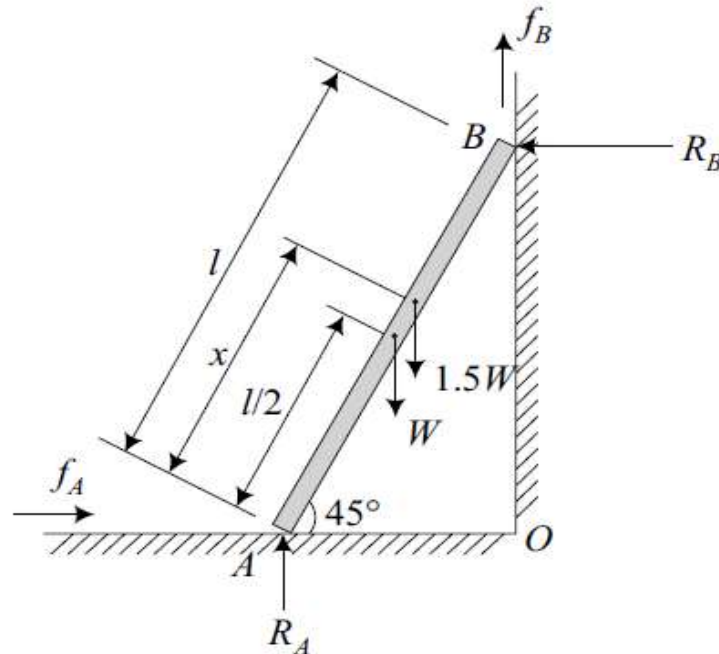
$$\mu = \frac{R_B}{N} = \frac{10}{20} = \mathbf{0.5} \quad \mathbf{Ans.}$$

INTRODUCTION TO FRICTION

LADDER FRICTION

Q.9. A ladder of length ' l ' rests against a wall, the angle of inclination being 45° . If the coefficient of friction between the ladder and the ground and that between the ladder and the wall is 0.5 each, what will be the maximum distance on ladder to which a man whose weight is 1.5 times the weight of the ladder may ascend before the ladder begins to slide?

Solution: Draw free body diagram as shown and apply equilibrium conditions.



INTRODUCTION TO FRICTION

LADDER FRICTION

$$(i) \quad \Sigma F_x = 0$$

$$f_A = R_B = \mu N = \mu R_A \\ = 0.5 R_A$$

\therefore

$$R_A = 2 R_B$$

$$(ii) \quad \Sigma F_y = 0$$

$$R_A + f_B = W + 1.5 W = 2 W$$

$$R_A = 2 R_B$$

$$f_B = \mu R_B = 0.5 R_B$$

\therefore

$$2 R_B + 0.5 R_B = 2 W$$

$$R_B = \frac{2}{2.5} W = 0.8 W$$

\therefore

$$f_B = 0.5 R_B = 0.4 W$$

$$(iii) \quad \Sigma M_A = 0$$

$$W \times \frac{l}{2} \cos 45^\circ + 1.5 W x \cos 45^\circ = R_B \times l \sin 45^\circ + f_B l \cos 45^\circ \\ = 0.8 W l \sin 45^\circ + 0.4 W l \cos 45^\circ$$

$$0.353 W l + 1.06 W x = 0.5656 W l + 0.2828 W l$$

$$1.06 x = (0.5656 + 0.2828 - 0.353) l = 0.49544 l$$

\therefore

$$\frac{x}{l} = \frac{0.49544}{1.06} = 0.467$$

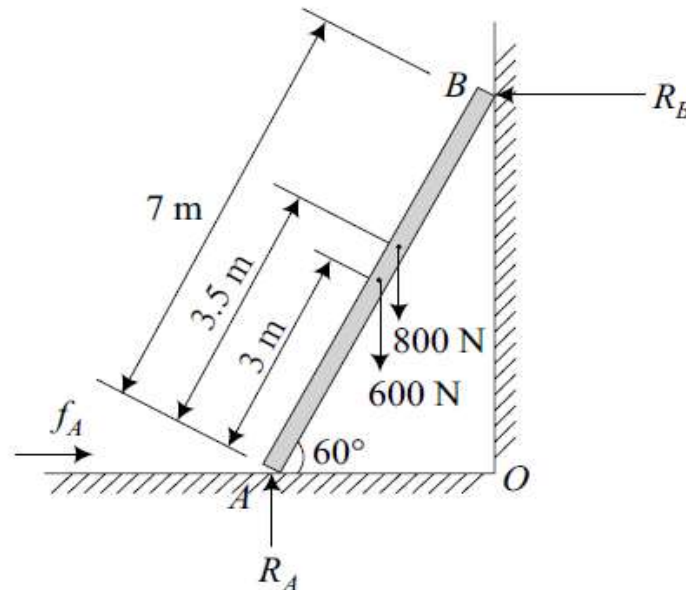
The man can ascend 46.7% of ladder length.

INTRODUCTION TO FRICTION

LADDER FRICTION

Q.10. A uniform ladder of weight 800 N and length 7 m rests on a horizontal ground and leans against a smooth vertical wall. The angle made by the ladder with the horizontal is 60° . When a man of weight 600 N stands on the ladder 4 m from the top of the ladder, the ladder is at the point of slipping. Determine the coefficient of friction between the ladder and the floor.

Solution: Draw free body diagram of the ladder and apply equilibrium conditions.



INTRODUCTION TO FRICTION

LADDER FRICTION

$$(i) \quad \Sigma F_x = 0$$

$$f_A = R_B$$

$$(ii) \quad \Sigma F_y = 0$$

$$R_A = 800 + 600 = 1400 \text{ N}$$

$$\begin{aligned} \therefore f_A &= R_B = \mu N = \mu R_A \\ &= 1400\mu \end{aligned}$$

$$(iii) \quad \Sigma M_B = 0$$

$$R_A \times 7 \cos 60^\circ = f_A \times 7 \sin 60^\circ + 800 \times 3.5 \cos 60^\circ + 600 \times 3 \cos 60^\circ$$

$$1400 \times 7 \cos 60^\circ = 1400\mu \times 7 \sin 60^\circ + (800 \times 3.5 + 600 \times 3) \cos 60^\circ$$

$$\therefore \mu = \frac{1400 \times 7 \cos 60^\circ - 4600 \cos 60^\circ}{1400 \times 7 \sin 60^\circ}$$

$$= \frac{4900 - 2300}{8487} = \mathbf{0.3} \quad \mathbf{Ans.}$$