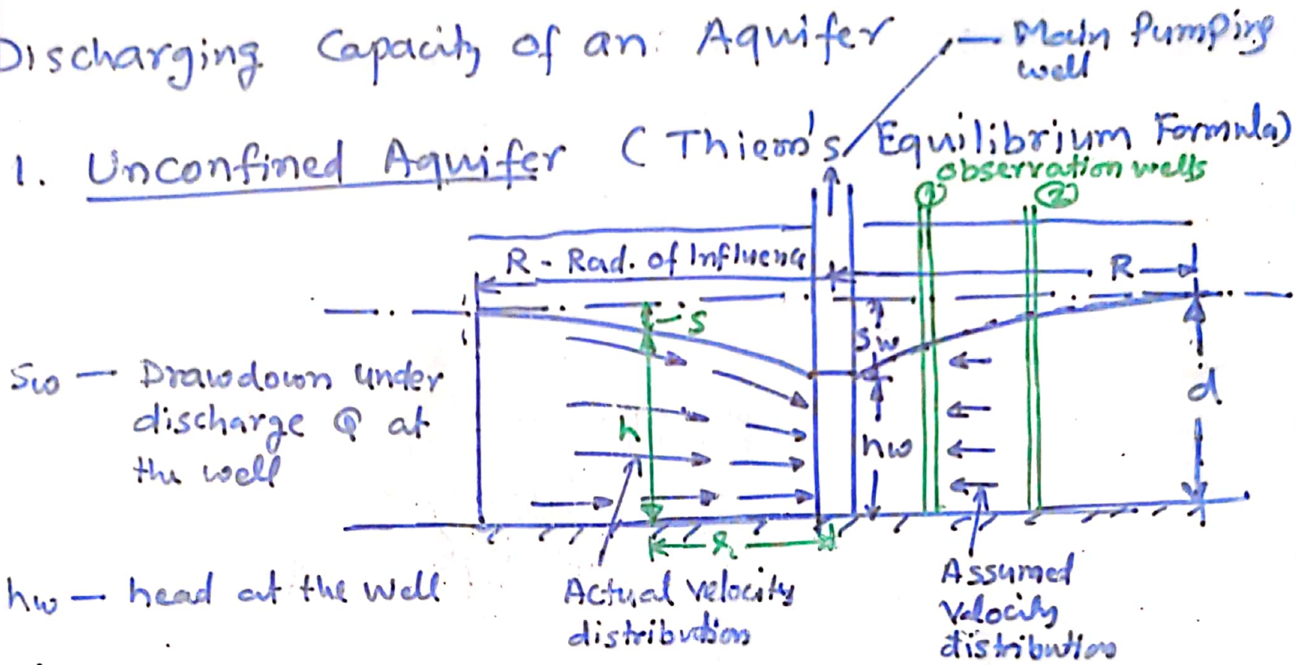
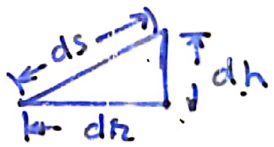
Discharging Capacity of an Aquifer

1. Unconfined Aquifer (Thiem's Equilibrium Formula)



From Darcy Law

$$Q = k\,i\,A$$



i = hydraulic gradient

$$i = \frac{dh}{ds}, \quad \text{but } ds \simeq dr$$

$$\therefore i = \frac{dh}{dr}$$

$$\therefore Q = k\frac{dh}{dr} \times 2\pi r h$$

$$\frac{dr}{r} = \frac{2\pi k}{Q} h\,dh$$

on intergrating

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k}{Q}\int_{h_1}^{h_2} h\,dh$$

$$\ln\frac{r_2}{r_1} = \frac{2\pi k}{Q}\frac{(h_2^2 - h_1^2)}{2}$$

$$\therefore \boxed{Q = \frac{\pi k (h_2^2 - h_1^2)}{\ln\frac{r_2}{r_1}}}$$

Now $(h_2^2 - h_1^2) = (h_2 + h_1)(h_2 - h_1)$

If $h_2 \simeq h_1 \simeq d$

$\therefore h_2 + h_1 \simeq 2d$

and $h_2 - h_1 = s_1 - s_2$

$$\therefore Q = \frac{\pi k \times 2d \times (s_1 - s_2)}{\ln \frac{r_2}{r_1}}$$

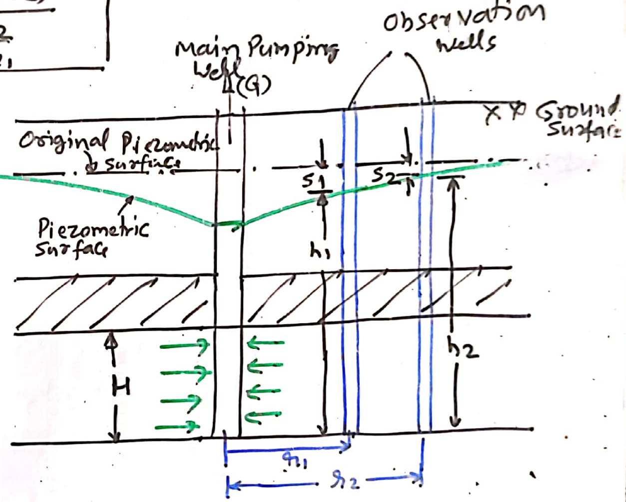
$kd = T$ (Transmissibility)

$$\therefore \boxed{Q = \frac{2\pi T (s_1 - s_2)}{\ln \frac{r_2}{r_1}}}$$

Assumptions

1. Aquifer is fully homogeneous
2. Well is dug upto full depth
3. k is uniform
4. GWT is horizontal
5. Equilibrium condition is fully reached

2. Confined Aquifer



$Q = k\,i\,A$

$$Q = k \frac{dh}{dr} 2\pi r H$$

$$Q = 2\pi k H\, r \frac{dh}{dr}$$

$$\int_{r_1}^{r_2} \frac{dr}{r} = \int_{h_1}^{h_2} \frac{2\pi k H}{Q} dh$$

$$\ln \frac{r_2}{r_1} = \frac{2\pi k H}{Q} (h_2 - h_1)$$

$$\therefore Q = \frac{2\pi k H}{\ln \frac{r_2}{r_1}} \times (h_2 - h_1)$$

$kH = T$ and $h_2 - h_1 = s_1 - s_2$

$$\therefore Q = \frac{2\pi T (s_2 - s_1)}{\ln \frac{r_2}{r_1}}$$

s_1, s_2 are drawdowns at two observation wells

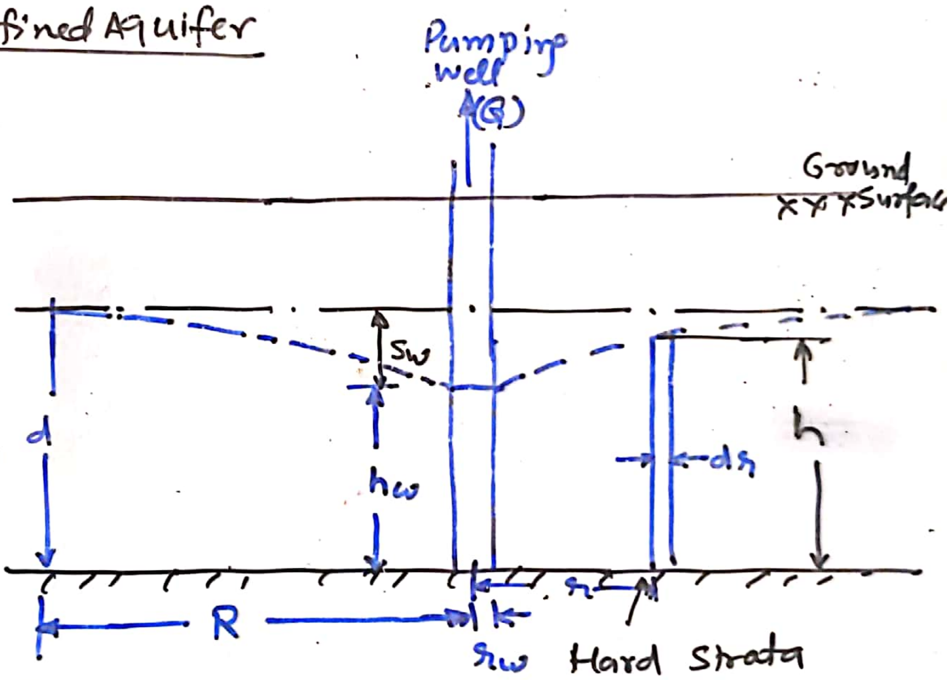
Dupit Formula

The above five conditions may not fully satisfied

In Dupit formula (i) no observation well is constructed.

(ii) main well is pumped out so as to get sufficient drawdown and then rate of pumping (Q) is so adjusted that so as to establish equilibrium condition ($Q_i = Q_o$) and water level in the well is constant

1. Gravity or unconfined Aquifer



$$Q = k\, i\, A$$

$$Q = k \frac{dh}{dr} \times 2\pi r h$$

$$\frac{dr}{r} = \frac{2\pi k}{Q} h\, dh$$

$$\int_{r_w}^{R} \frac{dr}{r} = \int_{h_w}^{d} \frac{2\pi k}{Q} h\, dh$$

$$\ln \frac{R}{r_w} = \frac{2\pi k}{Q} \frac{d^2 - h_w^2}{2}$$

$$Q = \frac{\pi k (d^2 - h_w^2)}{\ln \frac{R}{r_w}}$$

R = 150 m Slitcher

R = 300 m Tolman

However, $R = \phi(Q)$

and $R = cQ$

2. Pressure Well or Confined Aquifer

$$Q = \frac{2\pi k H (d - h_w)}{\ln \frac{R}{r_w}} = \frac{2\pi H\, S_w}{\left(\ln \frac{R}{r_w}\right)}$$